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POWRÓT

NUMERICAL DESCRIPTION OF WORKING PARAMETERS FOR HUMAN ELLIPTICAL HIP JOINT

OPIS NUMERYCZNY PARAMETRÓW PRACY DLA ELIPTYCZNEGO STAWU BIODROWEGO CZŁOWIEKA

Key-words:

Lubrication, human elliptical hip joint, working parameters

Słowa-kluczowe:

Smarowanie, eliptyczny staw biodrowy człowieka, parametry pracy

Summary

In 1971 Cathcart proposed the use of an elliptical head for the femoral endoprosthesis, based on his anatomical and clinical studies, which have proved that the normal femoral head is elliptical and not spherical. It was decided to use the an elliptical head of endoprosthesis because it improves the lubrication of acetabular cartilage and better distributes the loads.

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The numerical analysis of pressure, carrying capacity and compressive stresses are presented in this paper. It is taken into account the synovial unsymmetrical and stationary fluid flow in human elliptical hip joint. The paper shows the particular Reynolds equation for pressure distribution in hydrodynamic lubrication problem for human elliptical hip joint.

INTRODUCTION

Let us review what was happened in the presented below references. Dowson D. gives some basic biomechanics and biotribology of human synovial joints and joint replacements in the papers **[L. 4–5]**. Mow V.C. also presents the basic biomechanics of diarthrodinal human joints in the papers **[L. 6–7]**. Pipino F. and Molfetta L. describe the clinical and radiographic study of Cathcart elliptical orthocentric femoral prosthesis in the papers **[L. 8–9]**. These kind of elliptical prosthesis were firstly used in 1971. Wierzcholski K. was presented the mathematical and numerical model for working parameters in human hip joint for unsymmetrical and stationary fluid flow in the paper **[L. 10]** (pressure, capacity – spherical hip), for unsymmetrical and unstationary flow in the paper **[L. 11]** (pressure – spherical hip) and unsymmetrical and stationary flow caused not only by rotation but also by squeezing in the paper **[L. 12]** (pressure – spherical hip). The Autor of described a problem of working parameters in the human hip joint for stationary and unsymmetrical synovial fluid flow in the papers **[L. 2]** (pressure, capacity, stress – elliptical endoprosthesis) and **[L. 3]** (pressure, capacity, stress – elliptical hip hip joint, parameters: ω = 0,5 η = 0,25 for normal and ω = 0,4 η = 0,025 for pathological) and also for symmetrical fluid flow in the paper **[L. 1]** (pressure - elliptical hip joint).

The new element of this paper in compare with papers **[L. 1–3]**, **[L. 10–12]** and also **[L. 4–9]** is the numerical analysis of pressure, capacity force and stresses made for elliptical human hip joint.

The numerical analysis of pressure, carrying capacity and compressive stresses shown for synovial fluid flow in elliptical hip joint are presented in this paper. It is taken into account the stationary, unsymmetrical and isothermal flow of incompressible synovial fluid. Moreover only the circumferential the rotational motion of femoral head causes the fluid flow. In this model are considered the changeable gap height of hip joint and changeable synovial fluid viscosity. The gap height changes only in the limits of lubrication. The rotary motion of the elliptical femoral head it is possible only in the circumferential direction and also in the lubrication limits. It is neglected the roughness and irregularities occurring on the surface of the hip head and acetabulum. Fluid density is constant. It is taken the elliptical co-ordinates system (φ, r, ζ) – in circumference, radial and meridional direction respectively (**Fig. 1**). The components of the synovial fluid velocity vector **v** = [V_φ, V_r, V_ζ] are shown on **Fig. 2** and pressure localisation is presented on **Fig. 3**.

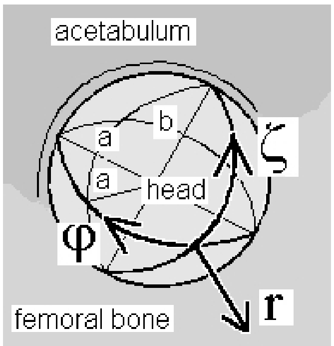

Fig. 1. Elliptical system
Rys.1. Układ eliptyczny

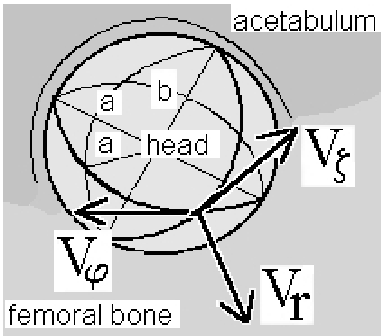

Fig. 2. Components of vector v
Rys. 2. Współrzędne wektora v

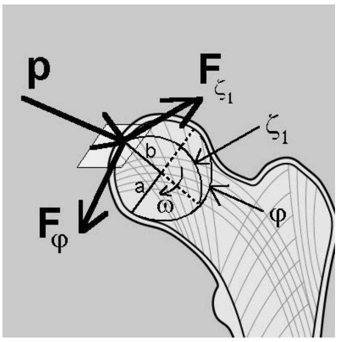

Fig. 3. Pressure localisation
Rys. 3 Lokalizacja ciśnienia

MATHEMATICAL DESCRIPTION

For axially unsymmetrical and stationary synovial fluid flow, pressure function depends on φ, ζ and dynamic viscosity η_p of synovial fluid depends on φ, r and ζ. The gap height ε may be a function of variable φ and

ζ. The elliptical surfaces in hip joint create a gap where is synovial fluid. The only circumferential rotation of elliptical femoral head with angular velocity ω causes a flow of synovial fluid in biobearing gap.

If we neglect the inertia and centrifugal forces, and then after layer boundary simplifications in general equations, we obtain Reynolds equation for pressure function $p(\varphi,\zeta)$ in the elliptical co-ordinate system. This equation has the following form:

$$\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon^3}{\eta_o}\frac{\partial p}{\partial\varphi}\right)+a\left[\sqrt{\cos^2\left(\frac{\zeta}{a}\right)+\frac{b^2}{a^2}\sin^2\left(\frac{\zeta}{a}\right)}\right]^{-1}\sin\left(\frac{\zeta}{a}\right)\times$$
$$\times\frac{\partial}{\partial\zeta}\left[\frac{\varepsilon^3 a}{\eta_o}\left[\sqrt{\cos^2\left(\frac{\zeta}{a}\right)+\frac{b^2}{a^2}\sin^2\left(\frac{\zeta}{a}\right)}\right]^{-1}\frac{\partial p}{\partial\zeta}\sin\left(\frac{\zeta}{a}\right)\right]=6a^2\omega\frac{\partial\varepsilon}{\partial\varphi}\sin^2\left(\frac{\zeta}{a}\right), \tag{1}$$

$$0<\varphi<2\pi c_1, \quad 0<c_1<1, \quad \frac{\pi a}{8}<\zeta<\frac{\pi a}{2}, \quad \zeta_1=\frac{\zeta}{a}, \quad a<b \tag{2}$$

where the symbol ω means the angular velocity of femoral head in [1/s], η_0 – the characteristic synovial fluid viscosity in [Pa·s], a and b – the radii of elliptical femoral head in [m], ε – the gap height in[m], p – the pressure in [Pa].

A centre of elliptical femoral head is in the point O(0,0,0) and centre of elliptical cartilage is in the point $O_1(x-\Delta\varepsilon_1, y-\Delta\varepsilon_2, z+\Delta\varepsilon_3)$ for hydrodynamic lubrication, which is caused only by head rotation.

On **Fig. 3** is shown some location of pressure $p(\varphi,\xi_1)$ on the head of hip joint with elliptical surface of lubrication $\Omega(\varphi,\xi)$ in [m^2]. In calculations is taken the values of atmospheric pressure p_{at} in [Pa]. Gap height calculated for elliptical femoral head has finally the following form:

$$\varepsilon(\varphi,\zeta_1)=\frac{a}{A}\sqrt{A^2\sin^2\zeta_1+B^2\cos^2\zeta_1}\cdot f(\varphi,\zeta_1)-\sqrt{a^2\sin^2\zeta_1+b^2\cos^2\zeta_1} \tag{3}$$

where $f(\varphi,\zeta_1)\equiv r_1$ is the positive solution of the equation:

$$[B^2a^2\sin^2\zeta_1+A^2b^2\cos^2\zeta_1]\,r_1^2+$$
$$-2[B^2a(\Delta\varepsilon_1\cos\varphi\sin\zeta_1+\Delta\varepsilon_2\sin\varphi\cos\zeta_1)-A^2b\Delta\varepsilon_3\cos\zeta_1]\,r_1+ \tag{4}$$

$$+B^2[\Delta\varepsilon_1^2+\Delta\varepsilon_2^2]+A^2\Delta\varepsilon_3^2-A^2B^2=0,$$

$$A\equiv a+D+\varepsilon_{min},\quad B\equiv b+D+\varepsilon_{min},\quad D\equiv\sqrt{\Delta\varepsilon_1^2+\Delta\varepsilon_2^2+\Delta\varepsilon_3^2} \tag{5}$$

where D means the eccentricity in [m] and r_1 is dimensionless radial direction in elliptical co-ordinate system and $\varepsilon(\varphi,\xi_1)$ is changeable height of the gap in [m].

Total carrying capacity force C_{tot} in [N], area of lubrication S [cm^2] and compressive stress in [N/cm^2] on elliptical femoral head of the hip joint are obtained from formulae:

$$C_{tot}\equiv\iint_{\sigma(\varphi,\zeta)}p(\varphi,\zeta)\,d\sigma(\varphi,\zeta), \tag{7}$$

$$S\equiv\frac{\pi}{2}\left[b^2+\frac{a^2b}{2\sqrt{b^2-a^2}}\ln\left(\frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}}\right)\right]\cos\left(\frac{\pi}{8}\right),\quad a<b, \tag{8}$$

$$\sigma\equiv\frac{C_{tot}}{S} \tag{9}$$

where symbol $\iint(\ldots)d\sigma$ means the surface integral determined on the head of hip joint surface and the symbol $d\sigma$ denotes area element in the double integral.

NUMERICAL ANALYSIS

In numerical analysis the symbols a, b mean the radii of elliptical head of hip joint in [cm], ω – the real angular velocity of the head in [1/s], η – the right values of synovial fluid viscosity in [Pa·s], p_{max} – the maximal calculated value of pressure in [Pa], ε_{min} – the minimum of the gap height in [μm], C_{tot} – the calculated total value of capacity in [N], S – the calculated surface region on femoral head in [cm^2] and σ – the calculated values of compressive stresses in [N/cm^2]. The symbols a, b, ω, η, ε_{min} mean the based parameters and C_{tot}, p_{max}, S, σ are calculated parameters. Moreover in the numerical analysis using the formulae (1)-(9) are taken $\Delta\varepsilon_1 = 2\ \mu m$, $\Delta\varepsilon_2 = 2\ \mu m$, $\Delta\varepsilon_3 = 2\ \mu m$.

Table 1. Parameters in numerical analysis of pressure for normal elliptical hip joint

	a [cm]	b [cm]	ω [1/s]	η [Pas]	p_{max} [Pa]	ε_{min} [μm]	C_{tot} [N]	S [cm^2]	$\sigma = C_{tot}/S$ [N/cm^2]
	2,5	1,02a = 2,550	0,75	0,2	$2{,}026\cdot10^6$	3	916	18,627	49,1759
	2,5	1,03a = 2,575	0,75	0,2	$2{,}012\cdot10^6$	3	927	18,872	49,1203
Fig. 4	2,5	1,04a = 2,600	0,75	0,2	$1{,}981\cdot10^6$	3	937	19,119	49,0088
Fig. 5	2,5	1,05a = 2,625	0,75	0,2	$1{,}998\cdot10^6$	3	945	19,368	48,7918

The results of pressure for normal elliptical hip joint are shown on **Fig. 4–5**.

Table 2. Parameters in analysis of pressure for pathological elliptical hip joint

	a [cm]	b [cm]	ω [1/s]	η [Pas]	p_{max} [Pa]	ε_{min} [μm]	C_{tot} [N]	S [cm^2]	$\sigma = C_{tot}/S$ [N/cm^2]
	2,4	1,02a = 2,448	0,25	0,25	$1{,}504\cdot10^6$	1	436	17,167	25,3975
	2,4	1,03a = 2,472	0,25	0,25	$1{,}480\cdot10^6$	1	439	17,393	25,2400
Fig.6	2,4	1,04a = 2,496	0,25	0,25	$1{,}457\cdot10^6$	1	441	17,621	25,0269
Fig.7	2,4	1,05a = 2,520	0,25	0,25	$1{,}433\cdot10^6$	1	443	17,849	24,8193

The results of pressure for pathological elliptical hip joint are shown on **Fig. 6-7**.

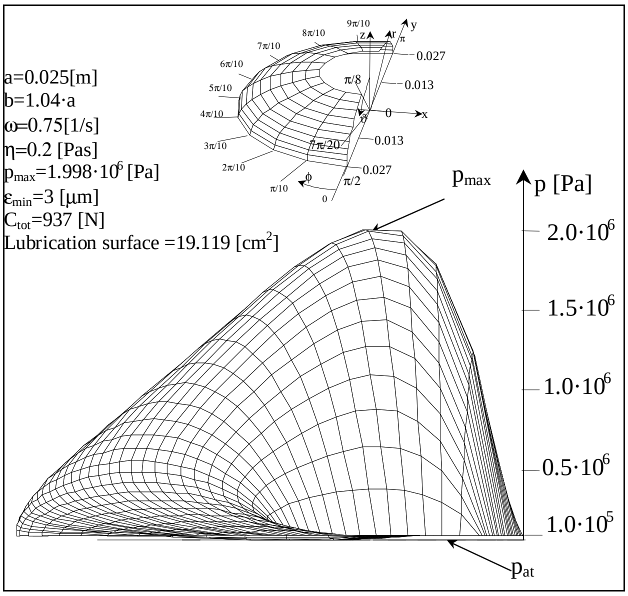

Fig. 4. Pressure distribution for normal elliptical hip joint: a = 0,025; b = 1,04a [m]
Rys. 4. Rozkład ciśnienia dla zdrowego eliptycznego stawu biodrowego: a = 0,025; b = 1,04a [m]

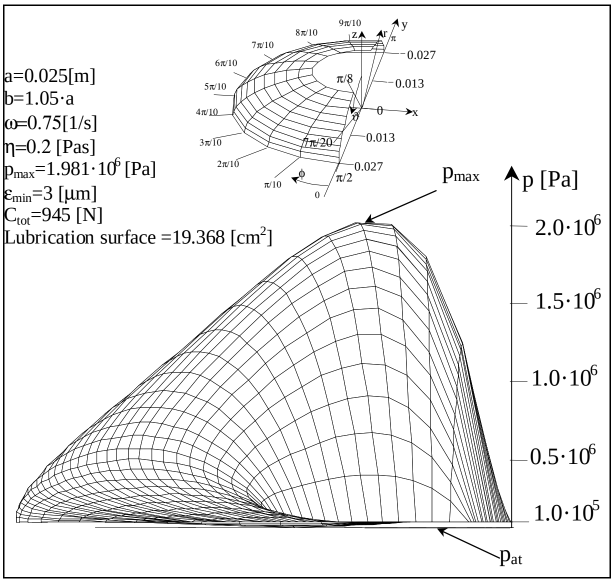

Fig. 5. Pressure distribution for normal elliptical hip joint: a = 0,025; b = 1,05a [m]

Rys. 5. Rozkład ciśnienia dla zdrowego eliptycznego stawu biodrowego: a = 0,025; b = 1,05a [m]

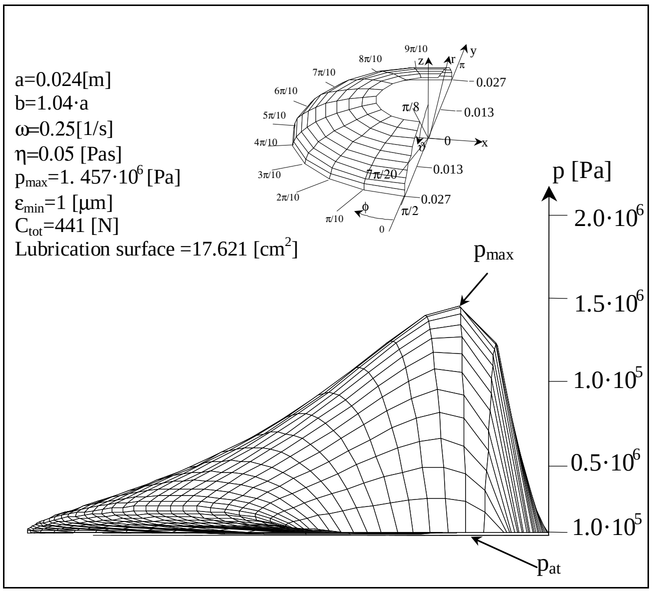

Fig. 6. Pressure distribution for pathological elliptical hip joint: a = 0,024; b = 1,04a [m]

Rys. 6. Rozkład ciśnienia dla patologicznego eliptycznego stawu biodrowego: a = 0,024; b = 1,04a [m]

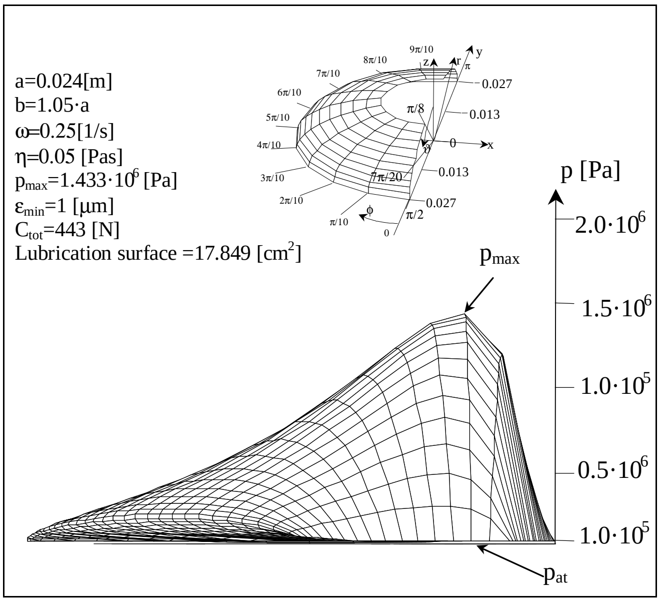

Fig. 7. Pressure distribution for pathological elliptical hip joint: a = 0,024; b = 1,05a [m]

Rys.7. Rozkład ciśnienia dla patologicznego eliptycznego stawu biodrowego: a = 0,024; b = 1,05a [m]

CONCLUSIONS

1. The analytical and numerical model of lubrication problem for human hip joint with elliptical co-operating surfaces allows to make the analysis of pressure, carrying capacity and compressive stresses.
2. For normal elliptical hip joint with based parameters a = 2,5 [cm], 2,550 [cm]<b<2,625 [cm], ω = 0,75 [1/s], η = 0,2 [Pa·s], ε_{min} = 3 [μm] obtained in the numerical analysis the following values for pressure from $2{,}026 \cdot 10^6$[Pa] to $1{,}998 \cdot 10^6$ [Pa], for total carrying capacity – from 916 [N] to 945 [N], for surface area from 18,627 [cm^2] to 19,368 [cm^2] and compressive stresses – from 49,1759 [N/cm^2] to 48,7918 [N/cm^2].
3. For pathological elliptical hip joint with based parameters a = 2,4 [cm], 2,448 [cm]<b<2,520 [cm], ω = 0,25 [1/s], η = 0,25 [Pa·s], ε_{min} = 1 [μm] obtained in the numerical analysis the following values for pressure from $1{,}504 \cdot 10^6$[Pa] to $1{,}433 \cdot 10^6$[Pa], for total carrying capacity – from 436 [N] to 443 [N], for surface area from 17,167 [cm^2] to

17,849 [cm^2] and compressive stresses – from 25,3975 [N/cm^2] to 24,8193 [N/cm^2].

4. The pressure, carrying capacity forces and compressive stresses values obtained for normal elliptical hip joint are smaller in compare with pressure, carrying capacity and compressive stresses values for pathological hip joint.

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Streszczenie

Z wieloletnich badań i obserwacji klinicznych wynika, że głowa stawu biodrowego człowieka ma raczej kształt elipsoidalny niż sferyczny. Fakt ten uzasadnia stosowanie endoprotez tego stawu o zarysie głowy elipsoidalnym, jednak pod warunkiem zachowania własnej panewki *(acetabulum)*. Stosowanie takich protez rozpoczął Cathcart już w 1971 roku [8]-[10].

W pracy zostały obliczone parametry pracy stawu biodrowego człowieka o eliptycznym zarysie współpracujących powierzchni. W modelu analityczno przyjęto izotermiczny, stacjonarny i osiowo-niesymetryczny przepływ cieczy synowialnej o nienewtonowskich własnościach. Po oszacowaniu równań podstawowych otrzymano uproszczony układ równań różniczkowych cząstkowych, z którego uzyskano równanie Reynoldsa wyznaczające ciśnienie. Równanie to zostało rozwiązane numerycznie. W hydrodynamicznym problemie smarowania stawu biodrowego wyznaczono wartość maksymalna ciśnienia, wartość naprężenia, wartość siły nośnej.

Dla stawu normalnego przy parametrach: a=2,5 cm i 2,550 < b < 2,625 cm, ω=0,75 1/s, η=0,2 Pa·s, ε_{min}=3 μm uzyskano wartość ciśnienia od $2{,}026\cdot10^6$ do $1{,}998\cdot10^6$ Pa, siły nośnej – od 916 do 945 N, naprężeń – od 49,1759 do 48,7918 N/cm^2 przy powierzchni od 18,627 do 19,368 cm^2.

Natomiast dla stawu chorego (osteoartrotycznego) przy parametrach: a=2,4 cm i 2,448 < b < 2,520 cm, ω=0,25 1/s, η=0,25 Pa·s, ε_{min}=1 μm uzyskano wartość ciśnienia od $1{,}504\cdot10^6$ do $1{,}433\cdot10^6$ Pa,

siły nośnej – od 436 do 443 N, naprężeń – od 25,3975 do 24,8193 N/cm^2 przy powierzchni od 17,167 do 17,849 cm^2.

Wyniki te mogą przyczynić się do lepszej diagnostyki i terapii ortopedycznej ludzkiego stawu biodrowego oraz jego protez.