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**THE DETERMINATION OF THE SYNOVIAL FLUID VISCOSITY ON
THE BASIS OF THE MEASURED FRICTION FORCES**

KEYWORDS

ABSTRACT

The author of the paper we present a method of the dynamic viscosity determination for a pathological synovial fluid using the experimental measured and calculated values of friction forces which occur in the biobearing gap.

INTRODUCTION

The viscosity of the synovial fluid with non-Newtonian properties was examined experimentally by D.Dowson and Cook (1990). Using the experimental values we obtained the approximation formulae for the dynamic values for small and large shear rates in the following form:

$$\eta_A \equiv \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + A \cdot \Theta} \approx \eta_0 - (\eta_0 - \eta_\infty)\Theta A + \dots \text{for } 0 < \Theta^2 B \ll 1 \quad (1)$$

And

$$\eta_p \equiv \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + A \cdot \Theta + B \cdot \Theta^2} \approx \eta_0 - (\eta_0 - \eta_\infty)\Theta A - (\eta_0 - \eta_\infty)\Theta^2 B + \dots \text{for other cases} \quad (2)$$

where η_∞ , η_0 means the dynamic viscosity value of synovial fluid for large and small shear rate values in Pa·s respectively. Moreover the symbols A and B denote the empirical coefficients which were obtained by Cooke and D.Dowson (1990). In the experimental way they obtained $A=1,88307$ s and $B=0,00458$ s² for the normal human joint and also $A=0,03349$ s and $B=0,00131$ s² for the pathological human joint. The shear rate has the form:

$$\Theta_0 \equiv O\left(\frac{V_0}{\varepsilon}\right), \Theta \equiv \frac{\partial v_1}{\partial \alpha_2} = \frac{\partial v_1^{(0)}}{\partial \alpha_2} + A \frac{\partial v_1^{(1)}}{\partial \alpha_2} + O(A^2) \quad (3)$$

where: ε -biobearing gap height, v_1 -circumference velocity of synovial fluid, $v_1^{(0)}$ -circumference velocity of normal synovial fluid, $v_1^{(1)}$ -changes of circumference velocity caused by the patho-

logical influences, α_2 -gap height direction. To describe the method of non-Newtonian pathological synovial fluid dynamic viscosity determination we must at first describe the friction forces occurring in the human biobearing gap. Total dimensional friction force has by virtue of calculation methods the following form [1, 2, 3, 4, 5]:

$$F_{R\Sigma} = \frac{bR^2\omega\eta_A}{\varepsilon} (F_{R1} + A^* \Delta F_{R1}) \quad (4)$$

η_A – the total non-Newtonian synovial fluid dynamic viscosity, F_{R1} – the friction force values for normal synovial fluid obtained from numerical calculations, ΔF_{R1} – the corrections of friction forces caused by the non-Newtonian synovial fluid properties obtained by virtue of numerical calculations, A^* – the dimensionless small parameter which describes the changes of dynamic viscosity values caused by the pathological non Newtonian influences, ω -angular velocity of bone head, R -radius of the bone head, b -biobearing length.

The geometry of biobearing joint shows Fig.1.

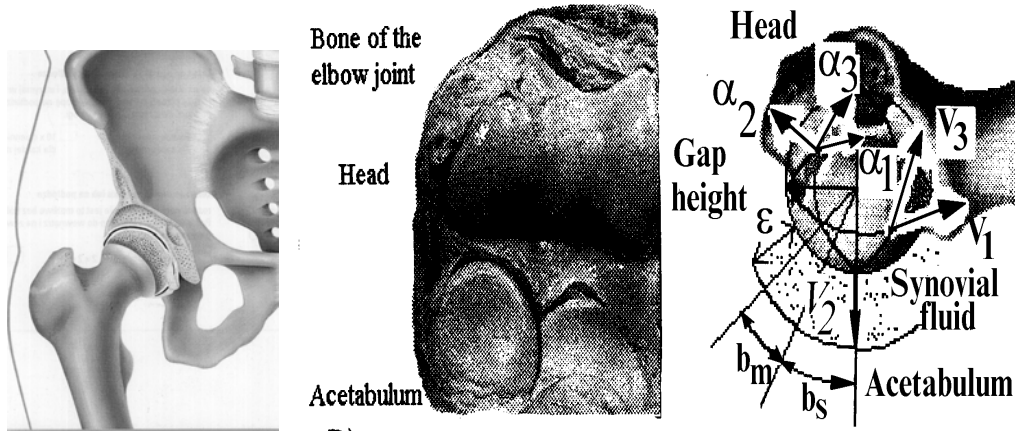


Fig. 1 Anatomical model of the human hip and elbow joint (articulatio cubiti) and geometry of the elbow spherical joint. Gap is shown in the enlarged scale

Small parameter is defined in following form [1]:

$$A^* = \frac{\eta_0 - \eta_A}{\eta_0} = 1 - \frac{\eta_A}{\eta_0} \quad (5)$$

where $\frac{\eta_A}{\eta_0}$ - the ratio of total dynamic viscosity η_p for non-Newtonian and pathological synovial fluids to the dynamic oil viscosity η_0 for normal and Newtonian synovial fluid.

From the Eq. (5) follows that the small parameter is the ratio of dynamic viscosity variations caused by the non-Newtonian synovial fluid pathological properties to the dynamic viscosity of the normal synovial fluid.

We assume that the total dimensional friction force $F_{R\Sigma}$ is equal to the total measured friction forces $F_R^{(A)}$ i.e.

$$F_{R\Sigma} \equiv F_R^{(A)} \quad (6)$$

Putting equality (6) into the Eq.(4) we obtain equation from this follows:

$$\eta_A = \frac{F_R^{(A)} \varepsilon}{bR^2 \omega (F_{R1} + A^* \Delta F_{R1})} \quad (7)$$

It is easy to see that if $A=0$ i.e. we have the normal properties of synovial fluid, then small parameter tends to zero (see Eq.5) and form dependence (7) follows:

$$\eta_o = \frac{F_R^{(0)} \varepsilon}{bR^2 \omega F_{R1}} \quad (8)$$

where $F_R^{(0)}$ is the measured friction force for normal synovial fluid
 η_o – the dynamic viscosity for normal synovial fluid.

2. DETERMINATION OF DYNAMIC VISCOSITY OF PATHOLOGICAL SYNOVIAL FLUID

Substituting the dependence (5) into the Eq.(7) we obtain the algebraic quadratic equation with respect to the dynamic viscosity η_A of the pathological synovial fluid. This quadratic equation has the following solution:

$$\eta_A = \frac{\eta_o}{2} \left(1 + \frac{F_{R1}}{\Delta F_{R1}} \right) \mp \left\{ \left[\frac{\eta_o}{2} \left(1 + \frac{F_{R1}}{\Delta F_{R1}} \right) \right]^2 - \frac{\varepsilon \eta_o F_R^{(A)}}{bR^2 \omega \Delta F_{R1}} \right\}^{1/2} \quad (9)$$

We put the value η_o from the Eq.(8) to the right hand of the Eq.(9) and hence we obtain finally [2]:

$$\frac{\eta_A}{\eta_o} = \frac{1}{2} \left[\left(1 + \frac{F_{R1}}{\Delta F_{R1}} \right) - \sqrt{\left(1 + \frac{F_{R1}}{\Delta F_{R1}} \right)^2 - 4 \frac{F_R^{(A)}}{F_R^{(0)}} \frac{F_{R1}}{\Delta F_{R1}}} \right] \quad (10)$$

where: $\frac{F_{R1}}{\Delta F_{R1}}$ – the ratio of calculated friction forces for the normal synovial fluid to its corrections caused by the pathological synovial fluid properties, $\frac{F_R^{(A)}}{F_R^{(0)}}$ – the ratio of measured friction forces for the normal synovial fluid to the friction forces for pathological synovial fluid properties.

If the ratio $\frac{F_R^{(A)}}{F_R^{(0)}}$ tends to unity for $A \rightarrow 0$, then the ratio of viscosity $\frac{\eta_A}{\eta_o}$ in the Eq.(10) tends to unity too. This case valid for the normal synovial fluid properties.

The formula (10) gives positive values for various practical real values of the ratio $\frac{F_{R1}}{\Delta F_{R1}}$.

3.DETERMINATION OF THE RATIO OF CALCULATED FRICTION FORCES

To obtain the ratio of calculated friction forces for the normal synovial fluid to its corrections caused by the pathological synovial fluid properties we must at first determine the dependencies between coefficient A and dimensionless small parameter A^* .

From the formula (5) we have:

$$\eta_A = \eta_o - \eta_o A^* \quad (11)$$

From the formula (1) we obtain:

$$\eta_A = \eta_o - \eta_o \Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2} A \quad (12)$$

where:

$$\Delta\eta \equiv 1 - \frac{\eta_\infty}{\eta_o} \quad (13)$$

Equating together equations (11) and (12) we obtain:

$$A = A^* \frac{1}{\Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2}} \quad (14)$$

The friction force has the following form:

$$F = R \int_0^{\phi_o} \int_{b_m}^{b_s} \eta_A \frac{\partial v_1}{\partial \alpha_2} d\alpha_3 d\alpha_2 \quad (15)$$

for $0 \leq \alpha_1 \leq \phi_o$, $b_m \leq \alpha_3 \leq b_s$, ϕ_o -range of lubrication, v_1 is the synovial fluid velocity component in circumference α_1 direction, ϕ_o -range of lubrication in α_1 direction, b_m , b_s -ranges of lubrication in α_3 direction. We put Eq. (12) and (1) into (15) and we obtain:

$$F = R\eta_o \int_0^{\phi_o} \int_{b_m}^{b_s} (1 - A \Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2}) (\frac{\partial v_1^{(o)}}{\partial \alpha_2} \pm A \frac{\partial v_1^{(1)}}{\partial \alpha_2}) d\alpha_3 d\alpha_2 \quad (16)$$

Now we substitute dependence (14) into formula (16). Thus we obtain:

$$F = R\eta_o \int_0^{\phi_o} \int_{b_m}^{b_s} (1 - A^* \Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2} \frac{1}{\Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2}}) (\frac{\partial v_1^{(o)}}{\partial \alpha_2} \pm A^* \frac{\partial v_1^{(1)}}{\partial \alpha_2} \frac{1}{\Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2}}) d\alpha_3 d\alpha_2 \quad (17)$$

After the simplifications we obtain:

$$F = R\eta_o \int_0^{\phi_o} \int_{b_m}^{b_s} (1 - A^*) (\frac{\partial v_1^{(o)}}{\partial \alpha_2} \pm A^* \frac{\partial v_1^{(1)}}{\partial \alpha_2} \frac{1}{\Delta\eta \frac{\partial v_1^{(o)}}{\partial \alpha_2}}) d\alpha_3 d\alpha_2 \equiv F_{R1} - A^* \Delta F_{R1} + 0(A^{*2}), \quad (18)$$

where friction forces on the sleeve surface are as follows:

$$F_{R1} = R\eta_o \left[\int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\partial v_1^{(o)}}{\partial \alpha_2} d\alpha_3 d\alpha_2 \right]_{\alpha_2 = \varepsilon}, \quad (19)$$

$$\Delta F_{R1} = R\eta_0 \left[\int_0^{\phi_0} \int_{b_m}^{b_s} \frac{\partial v_1^{(0)}}{\partial \alpha_2} d\alpha_3 d\alpha_2 \right]_{\alpha_2=\varepsilon} - R\eta_0 \frac{1}{\Delta\eta} \left[\int_0^{\phi_0} \int_{b_m}^{b_s} \frac{\partial v_1^{(1)}}{\partial \alpha_2} d\alpha_3 d\alpha_2 \right]_{\alpha_2=\varepsilon} \quad (20)$$

The ratio of friction forces has the following form:

$$\frac{\Delta F_{R1}}{F_{R1}} = 1 - \frac{1}{\Delta\eta} \frac{\left[\int_0^{\phi_0} \int_{b_m}^{b_s} \frac{\partial v_1^{(1)}}{\partial \alpha_2} d\alpha_3 d\alpha_2 \right]_{\alpha_2=\varepsilon}}{\left[\int_0^{\phi_0} \int_{b_m}^{b_s} \frac{\partial v_1^{(0)}}{\partial \alpha_2} d\alpha_3 d\alpha_2 \right]_{\alpha_2=\varepsilon}} \quad (21)$$

T

he first derivatives of circumference synovial fluid velocities with respect to the gap height coordinate have the following form [4]:

$$\left(\frac{\partial v_1^{(0)}}{\partial \alpha_2} \right)_{\alpha_2=\varepsilon} = \left(-\frac{1}{2\eta_0} \frac{1}{h_1} \frac{\partial p}{\partial \alpha_1} \varepsilon^2 \frac{\partial}{\partial \alpha_2} s \cdot (1-s) + \omega h_1 \frac{\partial}{\partial \alpha_2} (1-s) \right)_{\alpha_2=\varepsilon} = \frac{\varepsilon}{2\eta_0} \frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} - \frac{\omega h_1}{\varepsilon} \quad (22)$$

$$\begin{aligned} \left(\frac{\partial v_1^{(1)}}{\partial \alpha_2} \right)_{\alpha_2=\varepsilon} &= -\frac{\kappa_1}{8\eta_\infty} \cdot \varepsilon \frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} \left\{ 2\omega h_1 \left[\frac{\partial}{\partial \alpha_2} s(1-s) \right]_{\alpha_2=\varepsilon} + \frac{\varepsilon^2}{3\eta_0} \left(\frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} \right) \left[\frac{\partial}{\partial \alpha_2} s(1-s)(1-2s) \right]_{\alpha_2=\varepsilon} \right\} = \\ &= \frac{1}{4} \frac{\kappa_1 \omega}{\eta_\infty} \frac{\partial p^{(0)}}{\partial \alpha_1} - \frac{1}{24} \frac{\kappa_1}{\eta_\infty \eta_0} \varepsilon^2 \left(\frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} \right)^2 \end{aligned} \quad (23)$$

where $s \equiv \frac{\alpha_2}{\varepsilon} < 1$.

Moreover we have:

$$\kappa_1 \equiv \frac{4(\eta_\infty^2 - \eta_0 \eta_\infty)}{\eta_0^2}, \quad -\frac{1}{50} \leq \kappa_1 \leq -\frac{1}{25}, \quad \frac{\kappa_1 \eta_0}{\eta_\infty \Delta\eta} = \frac{4(\eta_\infty^2 - \eta_0 \eta_\infty)}{\eta_0^2} \frac{\eta_\infty}{\eta_0} \frac{1}{1 - \frac{\eta_\infty}{\eta_0}} = -4. \quad (24)$$

Symbol h_1 denotes Lamé coefficient. Hence we obtain the following expressions:

$$\left(\frac{\partial v_1^{(1)}}{\partial \alpha_2} \right)_{\alpha_2=\varepsilon} = \frac{\frac{1}{4} \frac{\kappa_1 \omega}{\eta_\infty} \frac{\partial p^{(0)}}{\partial \alpha_1} - \frac{1}{24} \frac{\kappa_1}{\eta_\infty \eta_0} \varepsilon^2 \left(\frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} \right)^2}{\frac{\varepsilon}{2\eta_0} \frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} - \frac{\omega h_1}{\varepsilon}} = \frac{\frac{1}{2} \frac{\eta_0}{\eta_\infty} \frac{\kappa_1 \omega h_1}{\varepsilon} - \frac{1}{12} \frac{\varepsilon \kappa_1}{\eta_\infty} \left(\frac{1}{h_1} \frac{\partial p^{(0)}}{\partial \alpha_1} \right)}{1 - \frac{2\eta_0 \omega h_1^2}{\varepsilon^2} \left(\frac{\partial p^{(0)}}{\partial \alpha_1} \right)^{-1}} \quad (25)$$

We substitute expressions (25) and (22) into formula (21), hence the ratio of friction forces has the following form:

$$\begin{aligned}
\frac{\Delta F_{R1}}{F_{R1}} = & 1 - \frac{1}{\Delta \eta} \frac{\left[\frac{1}{2} \frac{\eta_o}{\eta_\infty} \kappa_1 \frac{\omega}{\varepsilon} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{h_1}{1 - \frac{2\eta_o \omega h_1^2}{\varepsilon^2} \left(\frac{\partial p^{(o)}}{\partial \alpha_1} \right)^{-1}} d\alpha_3 d\alpha_2 \right]}{\frac{\varepsilon}{2\eta_o} \int_0^{\phi_o} \int_{b_m}^{b_s} \left(\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega h_1}{\varepsilon^2} \right) d\alpha_3 d\alpha_2} + \\
& \frac{1}{12} \frac{\varepsilon}{\eta_\infty} \kappa_1 \frac{\omega}{\varepsilon} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1}}{1 - \frac{2\eta_o \omega h_1^2}{\varepsilon^2} \left(\frac{\partial p^{(o)}}{\partial \alpha_1} \right)^{-1}} d\alpha_3 d\alpha_2 \cdot \\
& + \frac{1}{\Delta \eta} \frac{\left[\frac{1}{2} \frac{\eta_o}{\eta_\infty} \kappa_1 \frac{\omega}{\varepsilon} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{h_1}{1 - \frac{2\eta_o \omega h_1^2}{\varepsilon^2} \left(\frac{\partial p^{(o)}}{\partial \alpha_1} \right)^{-1}} d\alpha_3 d\alpha_2 \right]}{\frac{\varepsilon}{2\eta_o} \int_0^{\phi_o} \int_{b_m}^{b_s} \left(\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega h_1}{\varepsilon^2} \right) d\alpha_3 d\alpha_2} .
\end{aligned} \tag{26}$$

Taking into account dependence (24) then the ratio of friction force (26) take the following form:

$$\begin{aligned}
\frac{\Delta F_{R1}}{F_{R1}} = & 1 + 4 \frac{\omega \eta_o}{\varepsilon^2} \frac{\int_0^{\phi_o} \int_{b_m}^{b_s} \frac{h_1}{1 - \frac{2\eta_o \omega h_1^2}{\varepsilon^2} \left(\frac{\partial p^{(o)}}{\partial \alpha_1} \right)^{-1}} d\alpha_3 d\alpha_2 \cdot \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1}}{1 - \frac{2\eta_o \omega h_1^2}{\varepsilon^2} \left(\frac{\partial p^{(o)}}{\partial \alpha_1} \right)^{-1}} d\alpha_3 d\alpha_2}{\int_0^{\phi_o} \int_{b_m}^{b_s} \left(\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega h_1}{\varepsilon^2} \right) d\alpha_3 d\alpha_2} - \frac{2}{3} \frac{\int_0^{\phi_o} \int_{b_m}^{b_s} \left(\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega h_1}{\varepsilon^2} \right) d\alpha_3 d\alpha_2}{\int_0^{\phi_o} \int_{b_m}^{b_s} \left(\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega h_1}{\varepsilon^2} \right) d\alpha_3 d\alpha_2} .
\end{aligned} \tag{27}$$

After mathematical performances and for $h_1=R$ - radius of the bone head, then the ratio of the friction forces (27) obtain the following form:

$$\begin{aligned}
\frac{\Delta F_{R1}}{F_{R1}} = & 1 + 4 \frac{\frac{1}{(b_s - b_m)} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega R^2}{\varepsilon^2} + \frac{2\eta_o \omega R^2}{\varepsilon^2}}{\frac{\partial p^{(o)}}{\partial \alpha_1} - \frac{2\eta_o \omega R^2}{\varepsilon^2}} d\alpha_3 d\alpha_2}{\frac{1}{2\eta_o \omega R (b_s - b_m)} \frac{\varepsilon^2}{R^2} \int_0^{\phi_o} \int_{b_m}^{b_s} \left(\frac{1}{h_1} \frac{\partial p^{(o)}}{\partial \alpha_1} \right) d\alpha_3 d\alpha_2 + 2 \frac{\varepsilon^2}{R^2} \frac{R^2}{\varepsilon^2} \phi_o} + \\
& \frac{-\varepsilon^2}{2\eta_o \omega R^3 (b_s - b_m)} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1} \frac{\partial p^{(o)}}{\partial \alpha_1}}{1 - \frac{\varepsilon^2}{2\eta_o \omega R^2} \left(\frac{\partial p^{(o)}}{\partial \alpha_1} \right)} d\alpha_3 d\alpha_2 \cdot \\
& - \frac{2}{3} \frac{\int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\partial p^{(o)}}{\partial \alpha_1} d\alpha_3 d\alpha_2}{\phi_o - \frac{\varepsilon^2}{2\eta_o \omega R^3 (b_s - b_m)} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\partial p^{(o)}}{\partial \alpha_1} d\alpha_3 d\alpha_2} .
\end{aligned} \tag{28}$$

Now we introduce following notations for changes of Sommerfeld number and changes of load:

$$\Delta S_o \equiv \frac{\frac{\varepsilon^2}{R^2} \Delta W}{\eta_o \omega R (b_s - b_m)}, \quad \Delta W \equiv R \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{\partial p^{(o)}}{\partial \alpha_1} d\alpha_3 d\alpha_2 \quad (29)$$

then formula (28) has the following form:

$$\begin{aligned} \frac{\Delta F_{R1}}{F_{R1}} = & 1 + \frac{4}{\Delta S_o - 2\phi_o} \left[\phi_o - \frac{1}{(b_s - b_m)} \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{1}{1 - \frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1}} d\alpha_3 d\alpha_2 \right] + \quad (30) \\ & - \frac{4}{3(2\phi_o - \Delta S_o)} \frac{1}{(b_s - b_m)} \left[\int_0^{\phi_o} \int_{b_m}^{b_s} \left(1 + \frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1} \right) d\alpha_3 d\alpha_2 - \int_0^{\phi_o} \int_{b_m}^{b_s} \frac{1}{1 - \frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1}} d\alpha_3 d\alpha_2 \right] \end{aligned}$$

For the assumption:

$$\left| \frac{1}{1 - \frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1}} \right| \ll 1 \quad \text{and} \quad \frac{1}{1 - \frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1}} \cong 1 + \frac{\varepsilon^2}{2\eta_o \omega R^2} \frac{\partial p^{(o)}}{\partial \alpha_1} \quad (31)$$

then we obtain finally:

$$\frac{\Delta F_{R1}}{F_{R1}} = -\frac{\Delta S_o + 2\phi_o}{\Delta S_o - 2\phi_o}, \quad \frac{F_{R1}}{\Delta F_{R1}} = -\frac{\Delta S_o - 2\phi_o}{\Delta S_o + 2\phi_o}, \quad 1 + \frac{F_{R1}}{\Delta F_{R1}} = \frac{4\phi_o}{\Delta S_o + 2\phi_o}. \quad (32)$$

Hence the ratio of calculated friction forces gives the ratio of viscosity (10) in following form:

$$\frac{\eta_A}{\eta_o} = \frac{2\phi_o}{\Delta S_o + 2\phi_o} \left[1 \pm \sqrt{1 + \left[\left(\frac{\Delta S_o}{2} \right)^2 - \phi_o \right] \frac{F_R^{(A)}}{F_R^0}} \right] \quad (33)$$

4. NUMERICAL SIMULATIONS

We perform the numerical calculations of formula (10) using computer PC Pentium II-233MHz for the various calculated friction forces and for the anticipated ratio $\frac{F_R^{(A)}}{F_R^{(0)}}$ of measured friction forces. The numerical results are shown in the Figure 2 and Figure 3.

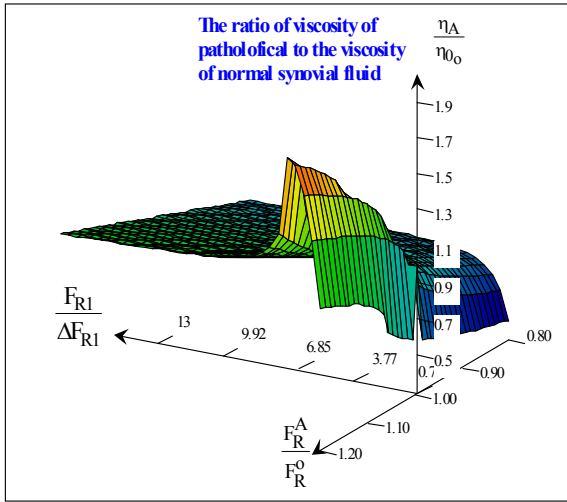
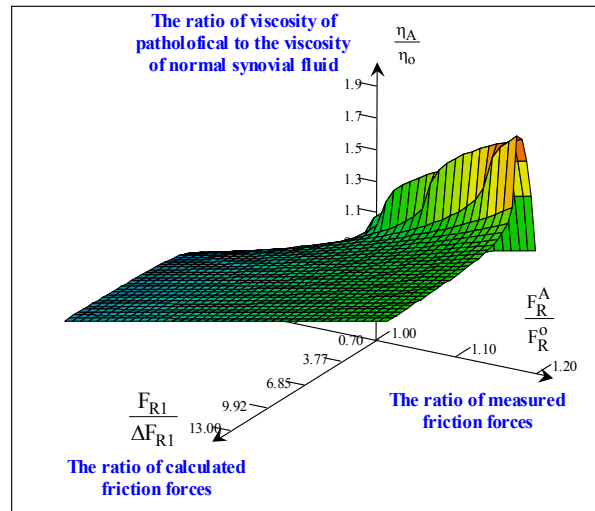


Fig. 2 Numerical dependencies between the dimensionless synovial fluid viscosity in the one hand and the ratio of calculated friction forces and anticipated ratio of measured friction forces in the other hand (first view)

Fig. 3 Numerical dependencies between the dimensionless synovial fluid viscosity in the one hand and the ratio of calculated friction forces and anticipated ratio of measured friction forces in the other hand (second view)



RESULTS

If the measured ratio $\frac{F_R^{(A)}}{F_R^{(O)}}$ greater than 1 is, i.e. the friction force for the non-Newtonian pathological synovial fluid is greater than friction force for the normal synovial fluid then oil dynamic viscosity of the pathological synovial fluid is greater than oil dynamic viscosity for the normal synovial fluid i.e. $\eta_A/\eta_o > 1$.

If the measured ratio $\frac{F_R^{(A)}}{F_R^{(O)}}$ smaller than 1 is, i.e. the friction force for the non Newtonian pathological synovial fluid is smaller than friction force for the normal synovial fluid, then oil dynamic viscosity of the pathological synovial fluid is smaller than oil dynamic viscosity for the normal synovial fluid i.e. $0 < \eta_A/\eta_o < 1$.

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WYZNACZANIE LEPKOŚCI DYNAMICZNEJ MAZI STAWOWEJ Z POMIARZONYCH SIŁ TARCIA

STRESZCZENIE:

Niniejsza praca przedstawia nową nieinwazyjną metodę wyznaczania lepkości dynamicznej chorej mazi stawowej w szczelinie biołożyska przy wykorzystaniu doświadczalnie pomierzonych oraz numerycznie obliczonych wartości sił tarcia, które występują w szczelinie funkcjonującego stawu człowieka. Bezpośredni pomiar lepkości mazi stawowej po wyjęciu jej z torebki stawowej nie jest zadawalający, ponieważ maź stawowa poza swoim naturalnym środowiskiem zmienia natychmiast swoją lepkość. Ponadto pobieranie mazi stawowej z funkcjonującego jeszcze stawu jest ze względu na zdrowie człowieka bardzo niewskazane.

K.Wierzcholski:Determination of the Synovial fluid viscosity from the measured friction forces

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