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POWRÓT

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THE INFLUENCE OF WALL'S POROSITYON THE PRESSURE DISTRIBUTION IN THE CURVILINEAR BEARING LUBRICATED BY POWER - LAW FLUID.

KEY WORDS

curvilinear thrust bearing, squeeze film, porous wall, power-law fluid.

ABSTRACT

The influence of porosity of the wall on the pressure distribution of a power-law fluid in a clearance of the curvilinear thrust bearing is considered. As a result one obtains the formulae expressing the pressure distribution. The example of a squeeze film between parallel disks is discussed in detail.

INTRODUCTION

Advances in technology and severe operational requirements of machines necessitated the development of improved lubricants for smooth and safe operation. Generally the viscosity of lubricating oils decreases with temperature. For operations under high speeds and heavy loads, oils containing high molecular-weight polymers as viscosity index improves are used to prevent viscosity variation with temperature. The increase in viscosity increases the load caring capacity of the modified lubricants.

Most lubricants are polymer solutions thus the characteristics of the bearings change when such rheological substances, known as non-Newtonian fluids, are used as lubricants. Many authors have studied the characteristics of various bearings by considering power-law [2], Bingham plastic [7], viscoplastic [4,7] and micropolar [3J models of lubricants.

The flows of Newtonian fluids in the clearance of a thrust bearing with impermeable surfaces have been examined theoretically and experimentally [5]. The bearing walls have been modelled as two disks, two conical or spherical surfaces. The more general case is established by the bearing formed by two surfaces of revolution [5-7].

Porous bearings have been widely used in industry for a long time. Basing on the Darcy model Morgan and Cameron [1] first presented theoretical research on these bearings.

The purpose of this study is to investigate the pressure distribution in the clearance of the thrust bearing formed by two surfaces of revolution, having parallel axes, shown in Fig.l; the lower one is connected with a porous layer. The analysis is based on the assumption that the porous matrix consists of a system of capillaries of very small radii restricting the floty of the lubricant through the matrix in only one direction.

ANALYSIS OF A FLUID FLOW IN THE BEARING CLEARANCE

The bearing floty configuration is shown in Fig. 1. The upper bound of a porous layer is described by function R(x) which denotes the radius of this bound. The fluid film thickness is given by function *h(x, t)* . An intrinsic curvilinear orthogonal co-ordinate system (x, Θ, *y)* is also depicted in Fig. l .

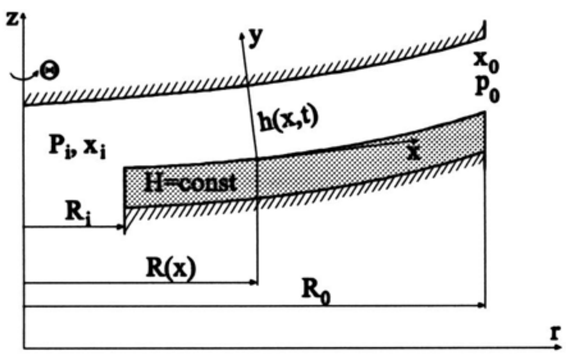

Fig. 1. Co-ordinate system in the bearing clearance
Rys. 1. Układ współrzędnych w szczelinie łożyska

By using the assumptions of hydrodynamic lubrication the equations of motion of a power-law fluid for axial symmetry one can present in the form [1,5,7]:

$$\frac{1}{R}\frac{\partial(R\upsilon_x)}{\partial x}+\frac{\partial\upsilon_y}{\partial y}=0, \tag{1}$$

$$\frac{\partial p}{\partial x}=m\frac{\partial}{\partial y}\left(\left|\frac{\partial\upsilon_x}{\partial y}\right|^{n-1}\frac{\partial\upsilon_x}{\partial y}\right), \tag{2}$$

$$\frac{\partial p}{\partial y}=0. \tag{3}$$

The problem statement is complete after specification of boundary conditions which are:

$$\upsilon_x(x,\theta,t)=0, \qquad \upsilon_x(x,h,t)=0, \tag{4}$$

$$\upsilon_y(x,\theta,t)=V, \qquad \upsilon_y(x,h,t)=-\frac{\partial h}{\partial t}; \tag{5}$$

$$p(x_i) = p_i\,, \qquad p(x_0) = p_0\,. \tag{6}$$

Here x_i denotes the inlet co-ordinate and x_o - the outlet co-ordinate.

Integrating Eq. (2) with respect to y in the interval $0 \le y \le h$ and determining the arbitrary constants from the boundary conditions (4) we obtain:

$$\upsilon_x = \frac{n}{n+1}\left(-\frac{1}{m}\frac{\partial p}{\partial x}\right)^{\frac{1}{n}}\left[\left(\frac{h}{2}\right)^{1+\frac{1}{n}} - \left|\frac{h}{2} - y\right|^{1+\frac{1}{n}}\right]. \tag{7}$$

Next, integrating the continuity equation (1) across the film thickness and taking finto account the boundary conditions (5) we have:

$$\frac{1}{R}\frac{\partial}{\partial x}\left[Rh^{2+\frac{1}{n}}\left(-\frac{1}{m}\frac{\partial p}{\partial x}\right)^{\frac{1}{n}}\right] = 2^{1+\frac{1}{n}}\left(\frac{2n+1}{n}\right)\left[\frac{\partial h}{\partial t} + V\right] \tag{8}$$

where V fis the velocity of lubricant on the upper bound of the porous matrix. This velocity may be determined as follows.

Consider that the porous matrix consists of a system of capillaries the axes of which are directed towards the film. The motion of the lubricant in a typical capillary fis governed by (see Fig.2):

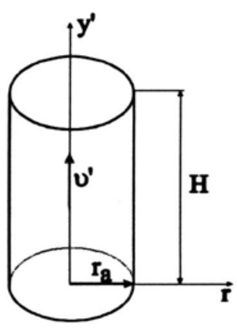

Fig.2. Flow of power-law lubricant in a thin capillary of porous layer
Rys.2. Przepływ potęgowego płynu w cienkiej kapilarze warstwy porowatej

$$\frac{dp'}{dy'} = \frac{m}{r}\frac{\partial}{\partial r}\left[r\left(-\frac{\partial \upsilon'}{\partial r}\right)^{n}\right] \tag{9}$$

since $\frac{\partial \upsilon'}{\partial r} \le 0$. Integrating Eq. (9) and using the boundary conditions

$$\frac{\partial \upsilon'}{\partial r} = 0 \quad for \quad r = 0, \quad \upsilon' = 0 \quad for \quad r = r_a \tag{10}$$

gives

$$\upsilon' = \frac{n r_a^{1+\frac{1}{n}}}{n+1}\left(-\frac{1}{2m}\frac{\partial p'}{\partial y'}\right)^{\frac{1}{n}}\left[1 - \left(\frac{r}{r_a}\right)^{1+\frac{1}{n}}\right]. \tag{11}$$

The flux may be defined as

$$Q' = 2\pi \int_0^{r_a} \upsilon' r dr = \frac{n\pi r_a^{3+\frac{1}{n}}}{2^{\frac{1}{n}}(3n+1)} \left(-\frac{1}{m} \frac{\partial p'}{\partial y'} \right) \tag{12}$$

where Q' is a constant as seen from the equation of continuity. The average velocity υ_0 of the lubricant can then be given as:

$$\upsilon_a = \frac{Q'}{\pi r_a^2} = \frac{n}{3n+1} \frac{r_a^{1+\frac{1}{n}}}{2^{\frac{1}{n}}} \left(-\frac{1}{m} \frac{dp'}{dy'} \right)^{\frac{1}{n}}. \tag{13}$$

This equation may be interpreted as a modified from of Darcy's law for power-law fluids. Assume now that the porous layer is homogeneous and isotropic and the floty in this layer satisfies the modified Darcy's law. Thus we have:

$$\upsilon'_x = \frac{n}{3n+1} \frac{r_a^{1+\frac{1}{n}}}{2^{\frac{1}{n}}} \left(-\frac{1}{m} \frac{\partial p'}{\partial x} \right)^{\frac{1}{n}}, \tag{14}$$

$$\upsilon'_y = \frac{n}{3n+1} \frac{r_a^{1+\frac{1}{n}}}{2^{\frac{1}{n}}} \left(-\frac{1}{m} \frac{\partial p'}{\partial y} \right)^{\frac{1}{n}}. \tag{15}$$

The equation of continuity in the porous region has the same form as Eq. (1):

$$\frac{1}{R} \frac{\partial (R\upsilon'_x)}{\partial x} + \frac{\partial \upsilon'_y}{\partial y} = 0. \tag{16}$$

Since the cross velocity component must be continuous at the porous wall-film interface, one obtains from Eqs (8) and (15) the modified Reynolds equation:

$$\frac{1}{R} \frac{\partial}{\partial x} \left[Rh^{2+\frac{1}{n}} \left(-\frac{1}{m} \frac{\partial p}{\partial x} \right)^{\frac{1}{n}} \right] = 2^{1+\frac{1}{n}} \left(\frac{2n+1}{n} \right) \left[\frac{\partial h}{\partial t} + \frac{n}{3n+1} \frac{r_a^{1+\frac{1}{n}}}{2^{\frac{1}{n}}} \left(-\frac{1}{m} \frac{\partial p'}{\partial y} \right)^{\frac{1}{n}} \Bigg|_{y=0} \right]. \tag{17}$$

By substituting Eqs (14) and (15) finto Eq. (16) one obtains the following equation for pressure distribution in the porous region:

$$\frac{1}{R} \frac{\partial}{\partial x} R \left(-\frac{1}{m} \frac{\partial p'}{\partial x} \right)^{\frac{1}{n}} + \frac{\partial}{\partial y} \left(-\frac{1}{m} \frac{\partial p'}{\partial y} \right)^{\frac{1}{n}} = 0. \tag{18}$$

Integrating this equation with respect to y over the porous layer and using the MorganCameron approximation [1] one obtains

$$\left(-\frac{1}{m}\frac{\partial p'}{\partial y}\right)^{\frac{1}{n}}\Bigg|_{y=0} = -\frac{H}{R}\frac{\partial}{\partial x}R\left(-\frac{1}{m}\frac{\partial p}{\partial x}\right)^{\frac{1}{n}}. \tag{19}$$

When Eq. (19) is substituted finto Eq. (17) the modified Reynolds equation takes the form:

$$\frac{1}{R}\frac{\partial}{\partial x}\left\{R\left[h^{2+\frac{1}{n}}+\beta(n)r_a^{1+\frac{1}{n}}H\right]\left(-\frac{1}{m}\frac{\partial p}{\partial x}\right)^{\frac{1}{n}}\right\}=[\alpha(n)]^{\frac{1}{n}}\frac{\partial h}{\partial t} \tag{20}$$

where

$$\beta(n)=\frac{2(2n+1)}{3n+1}, \qquad \alpha(n)=2^{n+1}\left(\frac{2n+1}{n}\right)^n. \tag{21}$$

SOLUTION. EXAMPLE OF APPLICATION

The form of solution of the Reynolds equation (20) depends on the bearing type. For the externally pressurised hydrostatic bearing $\left(\frac{\partial h}{\partial t}=0\right)$ and for the bearing with a squeeze film $\left(\frac{\partial p}{\partial x}\Big|_{x=0}=0\right)$ the solutions are, respectively:

$$p(x)=\frac{\left[A^{(n)}(x)-A_0^{(n)}\right]p_i-\left[A^{(n)}(x)-A_i^{(n)}\right]p_0}{A_i^{(n)}-A_0^{(n)}}, \tag{22}$$

$$p(x,t)=p_0+m\alpha(n)\left[S_0^{(n)}-S^{(n)}(x,t)\right] \tag{23}$$

where

$$A^{(n)}(x)=\int\frac{dx}{R^n\left[h^{2+\frac{1}{n}}+\beta(n)r_a^{1+\frac{1}{n}}H\right]^n}, \quad A_i^{(n)}=A^{(n)}(x_i), \quad A_0^{(n)}=A^{(n)}(x_0) \tag{24}$$

$$S^{(n)}(x,t)=\int\frac{\left[\int R\frac{\partial h}{\partial t}dx\right]^n}{R^n\left[h^{2+\frac{1}{n}}+\beta(n)r_a^{1+\frac{1}{n}}H\right]^n}dx, \quad S_0^{(n)}=S^{(n)}(x_0,t). \tag{25}$$

Introducing the following parameters:

$$\tilde{x} = \frac{x}{R_0}, \quad \tilde{R} = \frac{R}{R_0}, \quad \tilde{h} = \frac{h}{h_0}, \quad \frac{\partial \tilde{h}}{\partial \tilde{t}} = \frac{\partial h}{\partial t} / V_0, \quad K = \frac{r_a}{h_0},$$

$$\Phi = \frac{H}{h_0}, \quad \tilde{p} = \frac{(p - p_0) h_0^{2n+1}}{m R_0^{n+1} V_0^n}, \quad p^* = \frac{p}{p_0}, \quad \delta = \frac{p_i}{p_0} \tag{26}$$

and noting:

$$S^{(n)}(x,t) = \frac{R_0^{n+1} V_0^n}{h_0^{2n+1}} \tilde{S}^{(n)}(x,t), \quad A^{(n)}(x) = R_0^{-(n-1)} h_0^{-(2n+1)} \tilde{A}^{(n)}(\tilde{x}) \tag{27}$$

we may present Eqs (22) and (23) in the non dimensional forms:

$$p^*(\tilde{x}) = \frac{\left[\tilde{A}^{(n)}(\tilde{x}) - \tilde{A}_0^{(n)}\right]\delta - \left[\tilde{A}^{(n)}(\tilde{x}) - \tilde{A}_0^{(n)}\right]}{\tilde{A}_i^{(n)} - \tilde{A}_0^{(n)}}, \quad \tilde{p}(\tilde{x},t) = \alpha(n)\left[\tilde{S}_0^{(n)} - \tilde{S}^{(n)}(\tilde{x},t)\right] \quad (28),(29)$$

where

$$\tilde{A}^{(n)}(\tilde{x}) = \int \frac{d\tilde{x}}{\tilde{R}^n \left[\tilde{h}^{2+\frac{1}{n}} + \beta(n) K^{1+\frac{1}{n}} \Phi\right]^n}, \quad \tilde{S}^{(n)}(\tilde{x},t) = \int \frac{\left[\int \tilde{R} \frac{\partial \tilde{h}}{\partial \tilde{t}} d\tilde{x}\right]^n}{\tilde{R}^n \left[\tilde{h}^{2+\frac{1}{n}} + \beta(n) K^{1+\frac{1}{n}} \Phi\right]^n} dx. \tag{30}$$

Now consider, as an example, squeeze film porous bearing shown in Fig.3. The dimensionless pressure distribution is given by formula

$$\tilde{p}(\tilde{x},t) = \frac{2}{n+1}\left(\frac{2n+1}{n}\right)^n \frac{1 - \tilde{x}^{n+1}}{1 + \frac{2(2n+1)}{3n+1} K^{1+\frac{1}{n}} \Phi} \tag{31}$$

and it is depicted in Figs 4 and 5.

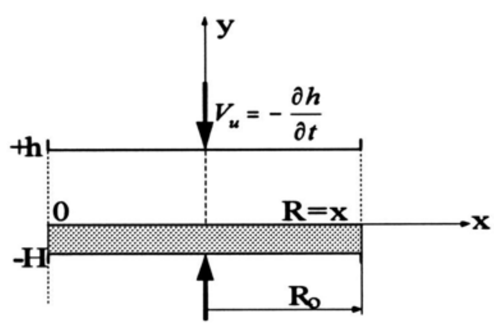

Fig.3. Squeeze film between two disks Rys.3. Wyciskany film między dwiema tarczami

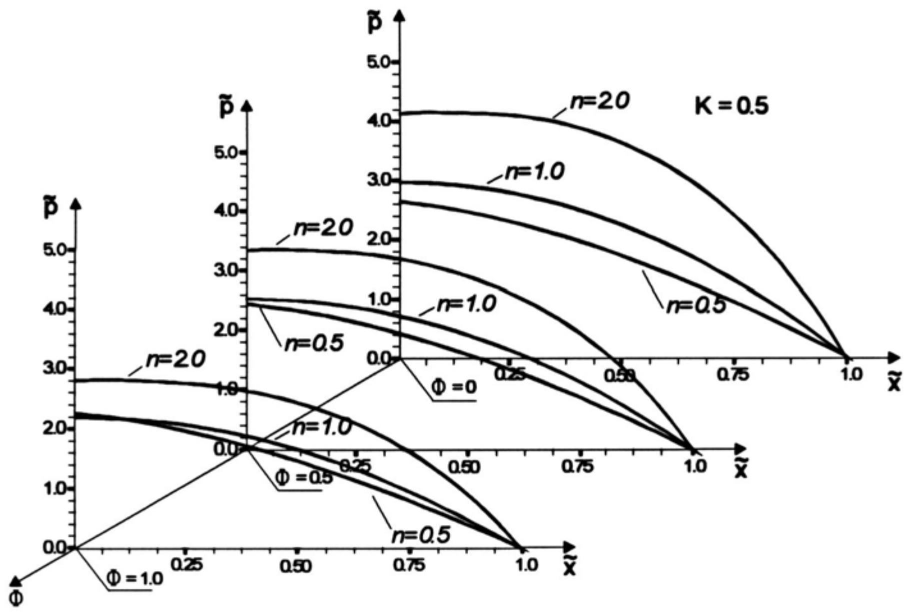

Fig.4. Dimensionless pressure distńbution for porosity *K = 0.5*

Rys.4. Bezwymiarowy rozkład ciśnienia dla porowatości *K = 0.5*

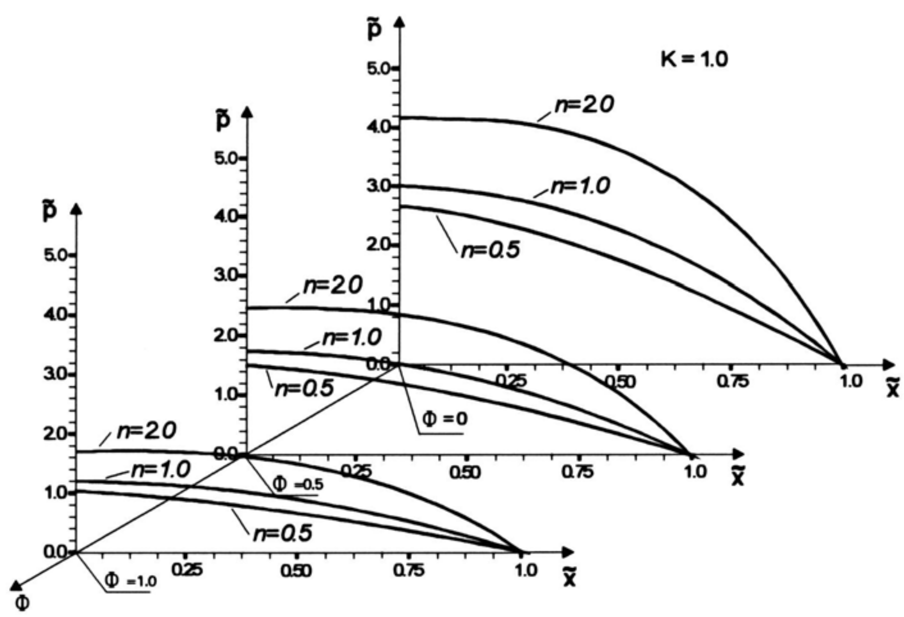

Fig.5. Dimensionless pressure distńbution for porosity *K = 1.0*

Rys.5. Bezwymiarowy rozkład ciśnienia dla porowatości *K = 1.0*

CONCLUSIONS

From the proceeding calculations and their graphical presentations for squeeze film bearing one may conclude that the pressure values decrease with the increase of porosity K and thickness θ of the porous layer. These values increase with the increase of the floty behaviour index n .

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WPŁYW POROWATOŚCI ŚCIANKI NA ROZKŁAD CIŚNIENIA W ŁOŻYSKU KRZYWOLINIOWYM WZDŁUŻNYM SMAROWANYM PŁYNEM POTĘGOWYM

Streszczenie

W pracy rozważany jest wpływ porowatości jednej ze ścianek ograniczających szczelinę wzdłużnego łożyska ślizgowego na rozkład ciśnienia w szczelinie. Do rozważań przyjęto model płynu potęgowego. Pomijając efekty bezwładnościowe wyprowadzono zmodyfikowaną postać równania różniczkowego Reynoldsa dla rozkładu ciśnienia. W równaniu tym uwzględniono wplyw porowatości ścianki poprzez wprowadzenie stosownych formuł przedstawiających zmodyfikowane prawo Darcy'ego dla przepływu płynu potęgowego w porowatym złożu.

Równanie Reynoldsa rozwiązano analitycznie uzyskując stosowne formuły dla rozkładu ciśnienia dla dwóch typów łożysk, mianowicie: zasilanych zewnętrznie i z wyciskanym filmem. Uzyskane rezultaty zilustrowano graficznie (Rys. 4 i 5) przedstawiając rozkłady ciśnienia w wyciskanym filmie między równoległymi płytkami kołowymi. Z przedstawionych formuł i wykresów wynika, że ciśnienie maleje ze wzrostem porowatości ścianki i ze wzrostem grubości warstwy porowatej oraz rośnie ze wzrostem wykładnika potęgowego n charakteryzującego płyn smarny

Recenzent: Prof. dr hab. inż. Jan Burcan