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POWRÓT

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MECHANICAL PARAMETERS OF THE CURVILINEAR THRUST BEARING LUBRICATED BY COUPLE STRESS FLUID

KEY WORDS

curvilinear thrust bearing, synovial joints, couple stress fluid.

ABSTRACT

Couple stresses affect lubrication problems when additives are used or a lubricant contains long-chain molecules. Such couple stresses may have a significant effect on the behaviour of bearings in practice. The influence of couple stresses witki and without rotational inertia on mechanical parameters of the curvilinear thrust bearing is considered. As an example the authors discuss the simulation of the squeeze-filin behaviour of a haman joint modelled as a hemispherical nonspinning bearing.

INTRODUCTION

The presence of small amounts of additives in a lubricant can improve bearing performance by enhancing the lubricant viscosity and thus producing an increase in the load capacity. They also reduce the coefficient of fi-fiction and increase the temperature range in which the bearing can operate.

The additives are long-chain organic compounds, e.g. the length of the polymer chain may be a million times the diameter of a water molecule. Thus couple stresses might be expected to appear in noticeable magnitudes in liquids containing additives witki these large molecules. These couple stresses may be significant particularly under lubrication conditions where thin films usually exist.
A number of theories of the microcontinuum have been postulated and applied [1,2,5]. One theory due to Stokes [1] allows for polar effects such as the presence of couple stresses and body couples.

This theory has been applied to the study of some simple lubrication problems (see e.g. [3,6]). However, its potential has not been explored fixlly. One of the possible application of couple stress fluid is its use to the study of the mechanismus of lubrication of synovial joints. In this paper Stokes theory [1] is applied to investigate the pressure distribution and the load of synovial joint modelled as a thrust bearing shown in Fig. 1.

EQUATIONS OF MOTION IN A THIN LAYER

The bearing floty configuration is shown in Fig. 1.

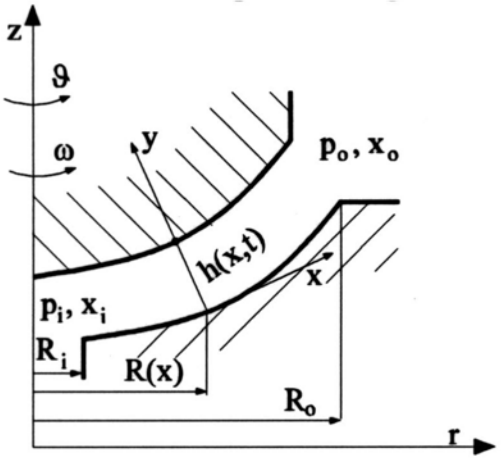


Fig. 1. Coordinate system in the bearing

Rys. 1. Układ współrzędnych w szczelinie łożyska

The fixed surface is described by function R(x) which denotes the radius of this surface. The fluid film thickness in the bearing clearance is described by function *h(x, t)* which denotes the distance between the fixed surface and the lower surface and the rotating surface, measured along a normal to the fixed surface. An intrinsic curvilinear orthogonal coordinate system x, ϑ, y linked with the fixed surface is presented in Fig. 1. The field equations for the motion of an incompressible fluid with couple stresses are (11:

$$div\overline{V} = 0, \tag{1}$$

$$\rho\frac{d\overline{V}}{dt} = -grad\,p + \mu\nabla^2\overline{V} - \eta\nabla^4\overline{V} \tag{2}$$

where $\overline{V}$ -is the velocity vector, p.- the hydrodynamic pressure, ρ - the density, μ -the shear viscosity and η is a nety material constant responsible for the couple stress property. The physical parameters of the floty are velocity components υ_x, υ_y, υ_z and pressure *p.*

After reduction allowed by assuming axial symmetry and that: *h(x, t)* « R(x) the governing differential equations are [4, 5]:

$$\frac{1}{R}\frac{\partial(R\upsilon_x)}{\partial x} + \frac{\partial\upsilon_y}{\partial y} = 0, \tag{3}$$

$$\mu\frac{\partial^2\upsilon_x}{\partial y^2} - \eta\frac{\partial^4\upsilon_x}{\partial y^4} = \frac{\partial p}{\partial x} - j\rho\upsilon_\vartheta^2\frac{R'}{R}, \tag{4}$$

$$\mu\frac{\partial^2\upsilon_\theta}{\partial y^2} - \eta\frac{\partial^4\upsilon_\theta}{\partial y^4} = 0, \tag{5}$$

$$\frac{\partial p}{\partial y} = 0. \tag{6}$$

Note that if j=0 the floty fis without rotational inertia.
The "prime" everywhere denotes derivation with respect to x. The problem statement fis complete after specification of boundary conditions which are:
- for the velocity components:

$$\upsilon_x(x,0,t) = \left.\frac{\partial^2 \upsilon_x}{\partial y^2}\right|_{y=0} = 0, \qquad \upsilon_x(x,h,t) = \left.\frac{\partial^2 \upsilon_x}{\partial y^2}\right|_{y=h} = 0, \tag{7}$$

$$\upsilon_\vartheta(x,0,t) = \left.\frac{\partial^2 \upsilon_x}{\partial y^2}\right|_{y=0} = 0, \qquad \upsilon_\vartheta(x,h,t) = R\omega, \left.\frac{\partial^2 \upsilon_\vartheta}{\partial y^2}\right|_{y=h} = 0, \tag{8}$$

$$\upsilon_y(x,0,t) = 0, \qquad \upsilon_y(x,h,t) = \frac{\partial h}{\partial t}, \tag{9}$$

- for the pressure:

$$p(x_i) = p_i, \qquad p(x_o) = p_o. \tag{10}$$

Here x; denotes the inlet coordinate and xa - the outlet coordinate.

SOLUTION OF THE EQUATIONS OF MOTION

Integating in sequence Eqs. (4) and (5) with respect to y in the interval $0 \le y \le h$ and determining the arbitrary constants from the boundary conditions (7) and (8) we obtain:

$$\upsilon_\vartheta = R\omega \frac{y}{h}, \tag{11}$$

$$\begin{aligned}\upsilon_x = &\left(\frac{1}{2\mu}\frac{\partial p}{\partial x} - \frac{j\rho l^2}{\mu}\frac{RR'\omega^2}{h^2}\right)\left[y^2 - hy + 2l^2\left(1 - ch\frac{y}{l} - \frac{1 - ch\, h/l}{sh\, h/l} sh\frac{y}{l}\right)\right] + \\ &+ \frac{j\rho RR'\omega^2}{\mu}\left[l^2\left(\frac{sh\, y/l}{sh\, h/l} - \frac{y}{h}\right) - \frac{1}{12h^2}\left(y^4 - h^3 y\right)\right]\end{aligned} \tag{12}$$

where

$$l^2 = \frac{\eta}{\mu}. \tag{13}$$

Next, integating the continuity equation (3) across the film thiclcness and taking finto account Eq. (12), and the boundary conditions (9) we have:

$$\frac{1}{R}\frac{\partial}{\partial x} R h^3 f(l,h)\left\{\frac{\partial p}{\partial x} - \frac{3j\rho RR'\omega^2}{10}\left[\frac{g(l,h)}{f(l,h)} - 40\left(\frac{l}{h}\right)^2\right]\right\} = 12\mu\frac{\partial h}{\partial t} \tag{14}$$

the modified Reynolds equation for the floty configuration considered. Here

$$f(l,h)=1-12\left(\frac{l}{h}\right)^{2}+24\left(\frac{l}{h}\right)^{3} th\left(\frac{h}{2l}\right), \tag{15}$$

$$g(l,h)=1+40\left(\frac{l}{h}\right)^{3}\left[cth\left(\frac{h}{l}\right)-\frac{h}{2l}\right]. \tag{16}$$

Integrating Eq. (14) and taking finto account the boundary condition (10) one obtains

$$p(x,t)=B(x,t)+\frac{\left[A(x,t)-A_0\right](p_i-B_i)-\left[A(x,t)-A_i\right](p_0-B_0)}{A_i-A_0} \tag{17}$$

where

$$A(x,t)=\int\frac{dx}{Rh^3 f(l,h)}, \qquad A_t(x,t)=\int R\frac{\partial h}{\partial t}dx,$$

$$D(x,t)=\int RR' F(l,h)dx, \qquad D_t(x,t)=\int\frac{A_t(x,t)}{Rh^3 f(l,h)}dx,$$

$$B(x,t)=\frac{3j\rho\omega^2}{10}D(x,t)+12\mu D_t(x,t), \qquad F(l,h)=\frac{g(l,h)}{f(l,h)}-40\left(\frac{l}{h}\right)^2, \tag{18}$$

$$A_i=A(x_i,t), \qquad A_0=A(x_0,t), \qquad B_i=B(x_i,t), \qquad B_0=B(x_0,t).$$

For bearings of the synovial joints the rotational inertia may be neglected *(j=0)* and the lubricant film fis purely squeezed. Therefore the solution of Eq. (14) satisfying the condition of symmetry.

$$\frac{\partial p}{\partial x}=0 \qquad \text{for} \qquad x=0 \tag{19}$$

fis given by the formula

$$p=p_0+12\mu\left[D_t(x,t)-D_{t0}\right]. \tag{20}$$

The load capacity of the bearing fis given by:

$$N=\pi R_i^{\,2}p_i+2\pi\int_{x_i}^{x_0}pR\cos\beta dx. \tag{21}$$

The sense of the angle β arises from Fig. 2.

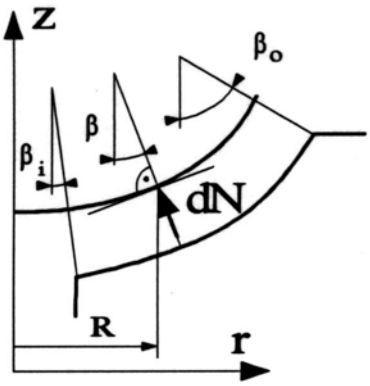


Fig. 2. Elementary load on bearing surface

Rys. 2. Elementarne obciążenie na powierzchni łożyska

EXAMPLE OF APPLICATION

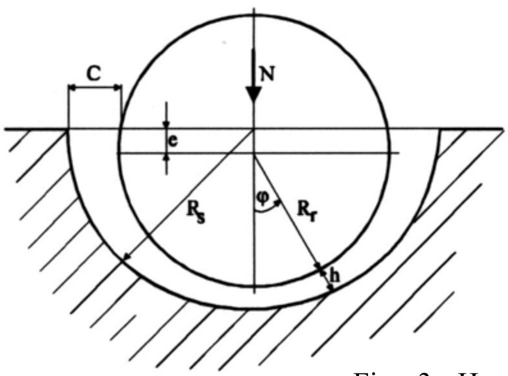
Let us consider the simulation of the squeeze-film behaviour of a human joint which geometry is shown in Fig. 3. The articulation of the joint is modelled as the case of a nonspinning spherical rotor approaching a hemispherical bearing with the velocity $\partial h / \partial t$.

Using the nondimensional parameters:

$$h^* = \frac{h}{C} = 1 - \varepsilon \cos\varphi , \qquad l^* = \frac{l}{C} \tag{22}$$

which are $h^* = 0(1)$, $l^* = 0(10^{-1})$ we can present the auxiliary function f (given by Eq. (15)) in a simpler form:

$$f\left(l^*, h^*\right) \approx 1 - 12\left(\frac{l^*}{h^*}\right)^2 . \tag{23}$$



$$C = R_s - R_r, \;\; h = C(1 - \varepsilon\cos\varphi)$$

$$\frac{\partial h}{\partial t} = -C\dot{\varepsilon}\cos\varphi, \;\; \varepsilon = \frac{e}{C}$$

$$R = R_r \sin\varphi, \;\; \varphi = \frac{x}{R_r}, \;\; \dot{\varepsilon} = \frac{d\varepsilon}{dt}$$

Fig. 3. Hemispherical bearing configuration
Rys. 3. Konfiguracja łożyska półkulistego

This form of f permits us to calculate the pressure distribution and the load capacity which are given by formulae (for $p_o = 0$):

$$p^* = \frac{pC^2}{\mu R_r^2 \dot{\varepsilon}} = \frac{3}{\varepsilon}\left\{\frac{1}{(1-\varepsilon\cos\varphi)^2} - 1 + 6l^*\left[\frac{1}{(1-\cos\varphi)^4 - 1}\right]\right\}, \tag{24}$$

$$N^* = \frac{NC^2}{\mu R_r^4 \dot{\varepsilon}} = \frac{6\pi}{\varepsilon^3}\left\{\frac{\varepsilon}{1-\varepsilon} - \frac{\varepsilon^2}{2} + \ln(1-\varepsilon) + 6l^{*2}\left(\frac{1}{3}\left[\frac{1}{(1-\varepsilon)^3} - 1\right] - \frac{1}{2}\left[\frac{1}{(1-\varepsilon)^2} - 1\right] - \frac{\varepsilon^2}{2}\right)\right\}. \quad (25)$$

RESULTS AND DISCUSSION

Fora Newtonian lubricant, the value of particie constant rl is equal to zero, and, thus, $1^* = 0$. As $1^* \to 0$, the functions p^* and N^* approache to the classical forms of the Newtonian lubricant case.

Theoretical pressure curves at eccentricity ratio ε = 0.4 for different values of couple stress parameter l* are shown in Fig. 4.

The effects of couple stresses are predominant for high values of 1* . The presence of couple stresses signifies an increase in the squeeze film pressure . The load capacity against eccentricity ratio s for different 1* is plotted in Fig . 5. The bearing lubricated with couple stress fluids yields a higher load capacity , especially for high values of ε.

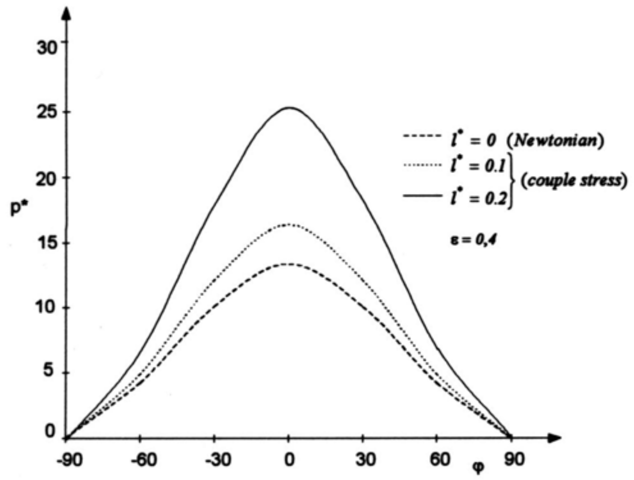


Fig. 4. Nondimensional pressure p^* as a fimction of φfor different 1*
Rys. 4. Bezwymiarowe ciśnienie p^* w funkcji φ dla różnych 1*

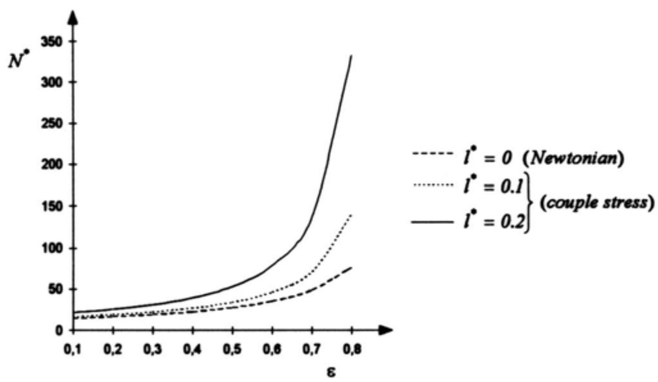


Fig. 5. Nondimensional load capacity N^* versus ε for different l^*
Rys. 5. Bezwymiarowa nośność N^* w zależności od ε dla różnych l^*

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PARAMETRY MECHANICZNE KRZYWOLINIOWEGO WZDLUŻNEGO ŁOŻYSKA ŚLIZGOWEGO SMAROWANEGO PŁYNEM Z NAPRĘŻENIAMI MOMENTOWYMI

Media smarne występujące w przegubach biologicznych charakteryzują się mikrowtrąceniami „usztywniającymi" film smarny ze wzrostem obciążenia. Jednym z częściej używanych modeli matematycznych płynu z mikrowtrąceniamijest model płynu polarnego, który może przenosić w trakcie ruchu naprężenia liniowe i momentowe. Celem przedstawionej pracy jest określenie parametrów mechanicznych krzywoliniowego wzdłużnego łożyska ślizgowego - modelującego łożysko biologiczne - smarowanego płynem polarnym. Jako szczególny prrykład rozpatrzono łożysko kuliste z wyciskanym filmem. Określając rzędy bezwymiarowej szczeliny h^* i bezwymiarowego współczynnika l^* charakteryzującego polarność plynu dokonano oszacowania współczynnika f w równaniu różniczkowym Reynoldsa, co pozwoliło na uzyskanie jego rozwiązania analitycznego . W efekcie otrzymano formuły określające rozkład ciśnienia i nośność łożyska . Wyniki przedstawione graficznie na Rys. 4 i 5 wskazują na wzrost obu parametrów ze wzrostem wartości l* .

Recenzent: Prof. dr hab. inż. Jan Burcan