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POWRÓT

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INTEGRAL APPROACHES FOR THE PRESSURE DISTRIBUTION OF POWER-LAW FLUID IN A CURVILINEAR BEARINGWITH SQUEEZE FILM

KEY WORDS

curvilinear squeeze film, integraf approaches, power-law fluid.

ABSTRACT

The influence of inertia effect on the pressure distribution of a power-law fluid in a cfearance of the curvilinear thrust bearing with squeeze film is considered. To solve this problem the boundary lager equations are used and expressed for the axially symmetric cale in a curvilinear coordinate system x, Θ, y connected with the median between surfaces limiting the bearing clearance. The method of integraf approaches is used to solve the boundary lager equations. As a result one obtains the formufae expressing the pressure distribution. The example of squeeze film between parallel dislcs is discussed in detaif.

INTRODUCTION

There has been considerable interest in recent years in the importance of fluid inertia effects in the field of hydrodynamic lubrication. The squeeze film problem is a particularly interesting one in this respect.

It is assumed in the classical theory of hydrodynamic lubrication that the effect of lubricant inertia is negligible in comparison to viscous forces. However, in lubricants with higher density and lower viscosity the effect of lubricant inertia becomes more important.

Many fluids of engineering interest appear to exhibit of non-Newtonian behaviour. To describe the rheological properties of such fluids the Ostwald - de Waele (or power-law) model is used.

This paper deals with the laminar squeezed floty of a power-law fluid in the clearance of smalt thickness between two curvilinear surfaces of revolution, having a common axis of symmetry, shown in Fig. 1. Basing on the method of integraf approaches as in [2,3] we have analysed the influence of inertia terms on the pressure distribution in the clearance.

BASIC EQUATIONS

The floty configuration is shown in Fig.l. The clearance thickness is described by the function *2h(x, t).* An intrinsic curvilinear orthogonal coordinate system (x, Θ, *y)* is also depicted in Fig. 1.

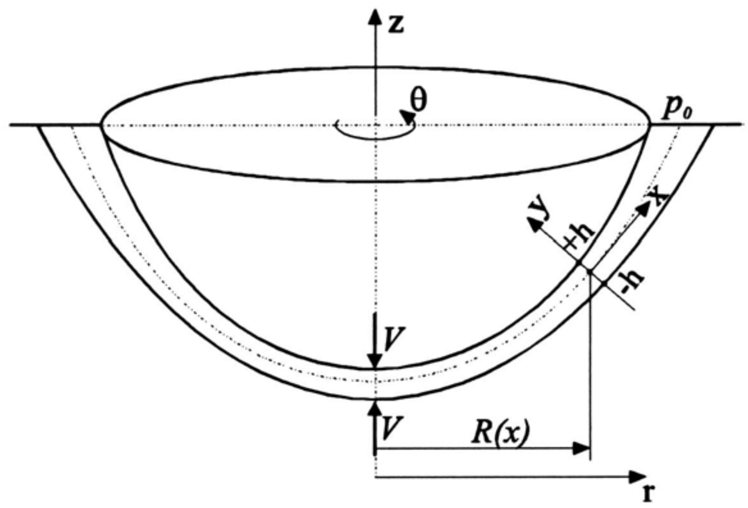

Fig. 1. Bearing clearance between curvilinear surfaces
Rys. 1. Szczelina łożyska między zakrzywionymi powierzchniami

By using the assumptions of hydrodynamic lubrication the equations of motion of a power-law fluid for axial symmetry one can present in the form [1=3]

$$\frac{1}{R}\frac{\partial(R\upsilon_x)}{\partial x}+\frac{\partial\upsilon_y}{\partial y}=0, \tag{1}$$

$$\rho\left(\upsilon_x\frac{\partial\upsilon_x}{\partial x}+\upsilon_y\frac{\partial\upsilon_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+m\frac{\partial}{\partial y}\left(\left|\frac{\partial\upsilon_x}{\partial y}\right|^{n-1}\frac{\partial\upsilon_x}{\partial y}\right), \tag{2}$$

$$\frac{\partial p}{\partial y}=0. \tag{3}$$

The boundary conditions for the velocity components υ_x and υ_y are the usual non-slip conditions stated as follows:

$$\begin{aligned} &\upsilon_x=0, \qquad \upsilon_y=V=\frac{\partial h}{\partial t} \quad \text{for } y=-h, \\ &\upsilon_x=0, \qquad \upsilon_y=V=-\frac{\partial h}{\partial t} \qquad \text{for } y=+h. \end{aligned} \tag{4}$$

The boundary conditions for the pressure are:

$$\left.\frac{\partial p}{\partial x}\right|_{x=0} = 0, \quad p = p_0 \quad \text{for } x = x_0 \tag{5}$$

We will use Eqs (1) =(3) to investigate the floty field by the method of integral approaches. Its concept consists in calculating first the solution of Eqs (1)-(3) in the Reynolds' approximation (without the inertia terms) and then in solving the complete set (with the inertia terms) by using the obtained approximation.

REYNOLDS' APPROXIMATION.

To find the Reynolds' approximation we replace Eq. (2) by the equation:

$$m\frac{\partial}{\partial y}\left(\left|\frac{\partial \upsilon_x}{\partial y}\right|^{n-1}\frac{\partial \upsilon_x}{\partial y}\right) = \frac{dp}{dx}; \tag{6}$$

here Eq. (3) was taken finto account. Integrating this equation

$$\upsilon_x(x,t) = \left(\frac{2n+1}{n+1}\right)\frac{Q}{4\pi R h^{2+1/n}}\left[h^{1+1/n} - |y|^{1+1/n}\right], \tag{7}$$

$$p(x,t) = p_0 + \frac{m}{(4\pi)^n}\left(\frac{2n+1}{n}\right)^n\left[S_0^{(n)} - S^{(n)}(x,t)\right] \tag{8}$$

where

$$Q = 4\pi\int R\frac{\partial h}{\partial t}dx, \qquad S^{(n)}(x,t) = \int\frac{Q^n}{R^n h^{2n+1}}dx, \quad S_0^{(n)} = S^{(n)}(x_0,t). \tag{9}$$

INTEGRAL APPROACHES

To analyse the influence of inertia terms of Eq. (2) on the pressure distribution in the slot we rearrange it to simplify its integration.

For this purpose we multiply Eq. (2) by ν_x^k and Eq. (1) by $\rho\nu_x^{k+1}/(k+1)$ and add the obtained expressions. As a result we have:

$$\frac{\rho}{k+1}\left[\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right)\upsilon_x^{k+2} + \frac{\partial}{\partial y}\left(\upsilon_x^{k+1}\upsilon_y\right)\right] = -\upsilon_x^k\frac{dp}{dx} + mn\upsilon_x^k\left|\frac{\partial \upsilon_x}{\partial y}\right|^{n-1}\frac{\partial^2 \upsilon_x}{\partial y^2}, \tag{10}$$

the prime denotes the differentiation with respect to x.

Integrating this equation across the clearance thickness, taking finto account boundary conditions (4) and introducing auxiliary notations:

$$J_{k-2}^{(1)} = \int_{-h}^{+h} \upsilon_x^k \left|\frac{\partial \upsilon_x}{\partial y}\right|^{n-1} \frac{\partial^2 \upsilon_x}{\partial y^2} dy, \qquad J_k^{(2)} = \int_{-h}^{+h} \upsilon_x^k dy,$$

$$J_{k+2}^{(3)} = \int_{-h}^{+h} \upsilon_x^{k+2} dy \tag{11}$$

we obtain the following relation:

$$\frac{\rho}{k+1}\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) J_{k+2}^{(3)} = -J_k^{(2)} \frac{dp}{dx} + mn\, J_{k-2}^{(1)}. \tag{12}$$

Thence we have

$$\frac{dp}{dx} = mn \frac{J_{k-2}^{(1)}}{J_k^{(2)}} - \frac{\rho}{k+1} \frac{\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) J_{k+2}^{(3)}}{J_k^{(2)}}. \tag{13}$$

To evaluate the pressure distribution effectively we must calculate the integrals $J_{k-2}^{(1)}, J_k^{(2)}, J_{k+2}^{(3)}$

Taking finto account the velocity component υ_x defined by Eq. (7) and assuming for calculation the following values of k:

k=0 and *k=1*

one obtains - after integrating Eq. (13) - the following formula for the pressure distribution:

$$p(x,t) = p_R(x,t) + \frac{\rho C_k^{(n)}}{16\pi^2}\left[T_0^{(k)} - T^{(k)}(x,t)\right] \tag{14}$$

where

$$T^{(k)}(x,t) = \int \frac{1}{h}\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) \frac{Q^2}{R^2 h} dx \quad \text{for } k = 0,$$

$$T^{(k)}(x,t) = \int \frac{R}{Q}\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) \frac{Q^3}{R^3 h^2} dx \quad \text{for } k = 1, \tag{15}$$

$$T_0^{(k)} = T^{(k)}(x_0, t),$$

and

k	0	1
$C_k^{(n)}$	$2\frac{(2n+1)}{(3n+2)}$	$3\frac{(2n+1)^2}{(3n+2)(4n+3)}$

(16)

Here $p_R(x, t)$ denotes the pressure distribution for Reynolds' approximation given by formula (8). If $n=1$ then all above formulae reduce to those obtained in [2] for the cale of Newtonian floty.

EXAMPLE OF APPLICATION

Equation (8) and (14) may be nondimensionalized by using the following parameters:

$$\tilde{x} = \frac{x}{R_0}, \qquad \tilde{R} = \frac{R}{R_0}, \qquad \tilde{h} = \frac{h}{h_0}, \qquad \frac{\partial \tilde{h}}{\partial \tilde{t}} = \frac{\partial h}{\partial t} / V_0,$$

$$\tilde{p} = \frac{2^{n+1}(p - p_0)h_0^{2n+1}}{mR_0^{n+1}V_0^n}, \qquad R_\lambda = \frac{2^{n+1}\rho h_0^{2n-1}V_0^{2-n}}{mR_0^{n-1}}; \tag{17}$$

here R_λ denotes the modified Reynolds number.
Then

$$S^{(n)}(x,t) = \frac{(4\pi)^n R_0^{n+1} V_0^n}{h_0^{2n+1}} \tilde{S}^{(n)}(\tilde{x},t), \quad T^{(k)}(x,t) = \frac{(4\pi)^2 R_0^2 V_0^2}{h_0^2} \tilde{T}^{(k)}(\tilde{x},t), \tag{18}$$

and the nondimensional formula for the pressure distribution takes the form

$$\tilde{p}(\tilde{x},t) = 2^{n+1}\left(\frac{2n+1}{n}\right)^n \left[\tilde{S}_0^{(n)} - \tilde{S}^{(n)}(\tilde{x},t)\right] + C_k^{(n)} R_\lambda \left[\tilde{T}_0^{(k)} - \tilde{T}^{(k)}(\tilde{x},t)\right]; \tag{19}$$

if $R_\lambda = 0$ this formula represents the pressure distribution for Reynolds' approximation. Noty consider, as an example, the squeeze film between two disks shown in Fig. 2.

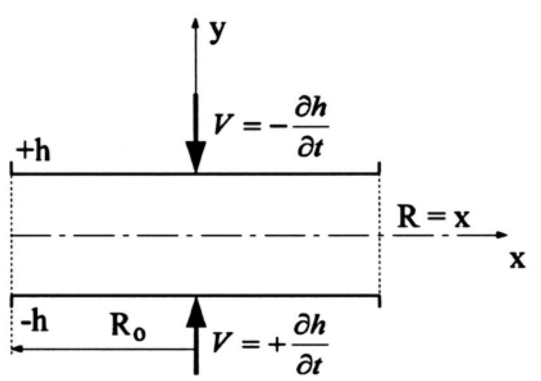

Fig. 2. Squeeze film between two disks
Rys. 2. Wyciskany film między dwiema tarczami

The dimensionless pressure distribution is given by formula:

$$\widetilde{p}(\widetilde{x},t) = \frac{2}{n+1}\left(\frac{2n+1}{n}\right)^n\left[1-\widetilde{x}^{n+1}\right] + \alpha(k)C_k^{(n)}R_\lambda\left[1-\widetilde{x}^2\right] \tag{20}$$

where

$$\alpha(0) = \frac{3}{8}, \qquad \alpha(1) = \frac{1}{2} \tag{21}$$

The gaphs of this distribution are depicted in Figs 3 and 4; $R_\lambda = 0$ represent the cases without inertia effects.

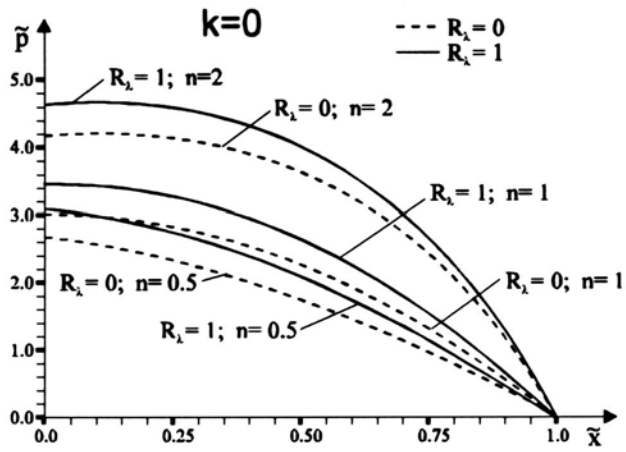

Fig. 3. Dimensionless pressure distribution for *k=0* (momentum integraf approach)
Rys. 3. Bezwymiarowy rozkład ciśnienia dla *k=0* (całkowe przybliżenie pędu)

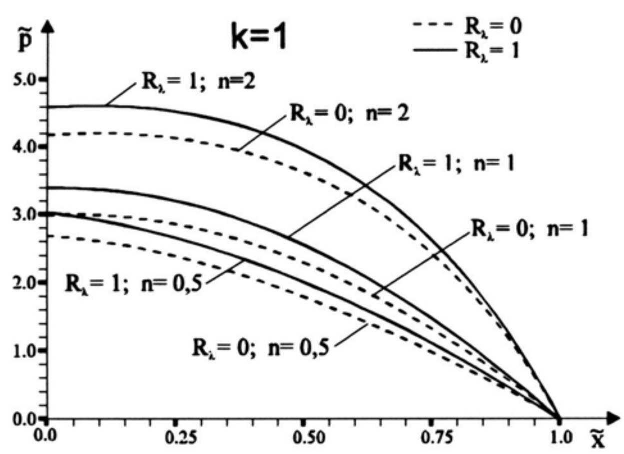

Fig. 4. Dimensionless pressure distribution for *k=1* (energy integraf approach)
Rys. 4. Bezwymiarowy rozkład ciśnienia dla *k=1* (całkowe przybliżenie energii)

CONCLUSIONS

Application of the method of integal approaches (momentum integal approach if *k=0* and energy integraf approach if *k=I)* to the study of the power-law fluid floty between two squeezed surfaces of revolution yields the formulae for the pressure distribution.

The formulae for the momentum integal approach (k=0) are identical with those obtained by the method of averaged inertia presented in [4,5].

It is seen from the graphs depicted in Figs 3 and 4 that the pressure increases witki the increase of the floty behaviour index n. The pressure increases also witki the increase of the modifled Reynolds number.

A comparison of the pressure distribution mada for the special cale of Newtonian floty *(n=I)* between parallel disks witki experimental data of Chen [2] indicates a satisfactory agreement for k=0 and a good agreement for *k=1* between the theoretical and experimental data.

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PRZYBLIŻENIE CAŁKOWE DLA ROZKŁADU CIŚNIENIA PŁYNU POTĘGOWEGO W KRZYWOLINIOWYM ŁOŻYSKU Z WYCISKANYM FILMEM

Streszczenie W pracy jest rozważany wpływ efektów bezwładnościowych na rozkład ciśnienia płynu potęgowego w szczelinie wzdłużnego łożyska ślizgowego z wyciskanym filmem. Do rozwiązania problemu użyto równań warstwy przyściennej, wyrażonych dla przypadku osiowej symetrii w układzie współrzędnych krzywoliniowych x, Θ, y związanym z powierzchnią środkową między powierzchniami ograniczającymi szczelinę łożyska. Aby rozwiązać równanie warstwy przyściennej zastosowano metodę przybliżeń całkowych. W rezultacie uzyskano formuły dla rozkładu ciśnienia w szczelinie zależne od rzędu przybliżenia całkowego wyrażonego pomocniczym wykładnikiem k, sens użycia tego wykładnika wynika z równania (10). Dla *k=0* otrzymujemy tzw. całkowe przybliżenie pędu, zaś dla *k=1* - całkowe przybliżenie energii. Rezultaty otrzymane dla całkowego przybliżenia pędu są identyczne z rezultatami uzyskanymi w pracach [4,5] metodą uśredniania składników bezwładnościowych w równaniu ruchu (2). Otrzymane wyniki zilustrowano przykładem rozkładu ciśnienia w wyciskanym filmie między równoległymi płytkami kołowymi. Wykresy ciśnień zależne od: rzędu przybliżenia k, wykładnika n charakteryzującego płyn potęgowy i zmodyfikowanej liczby Reynoldsa R_λ przedstawiono na Rys. 3 i 4. z przedstawionych formuł i wykresów wynika ogólny wniosek, że wzrost wartości n oraz R_λ prowadzi do wzrostu ciśnienia w wyciskanym filmie.

Recenzent: Prof. dr hab. inż. Jan Burcan