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POWRÓT

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MECHANICAL PARAMETERS FOR A MULTILOBE JOURNAL BEARING WITH A POROUS WALL

KEY WORDS

multilobe journal bearing, porous wali, Galerkin's method, Newtonian lubricant.

ABSTRACT

In this paper the authors present a solution to the problem of mechanical parameters of a multilobe journal bearing witki porous wall. The problem was solved by Galerkin's method after assuming that the bearing is lubricated witki an incompressible Newtonian fluid. The general form of solutions for particular cases of the bearing geometry are given.

INTRODUCTION

The steady laminar flows of a Newtonian fluid in a clearance of the bearing witki impermeable surfaces has been examined theoretically and experimentally [7, 8]. The bearing walls have been modelled as two cylindrical surfaces (for journal bearings) and two disks, two conical or spherical surfaces. The more general case is established by the bearing formed by two surfaces of revolution [6].

Porous bearings have been widety used in industry for a long time. Based on the Darcy model Morgan and Cameron [4] first presented theoretical research on these bearings. Recently, the solutions containing extended Darcy models of porous matrix are presented [2, 3].

The purpose of this study is to investigate the pressure distribution and the load capacity of the journal bearing formed by two cylindrical surfaces shown in Fig. 1. The Darcy model of porous matrix was assumed as in [11].

BEARING DESCRIPTION

Figure 1 shows the bearing configuration to be studied; this configuration is presented in a cylindrical coordinate system $\bar{x}$, R, Θ .

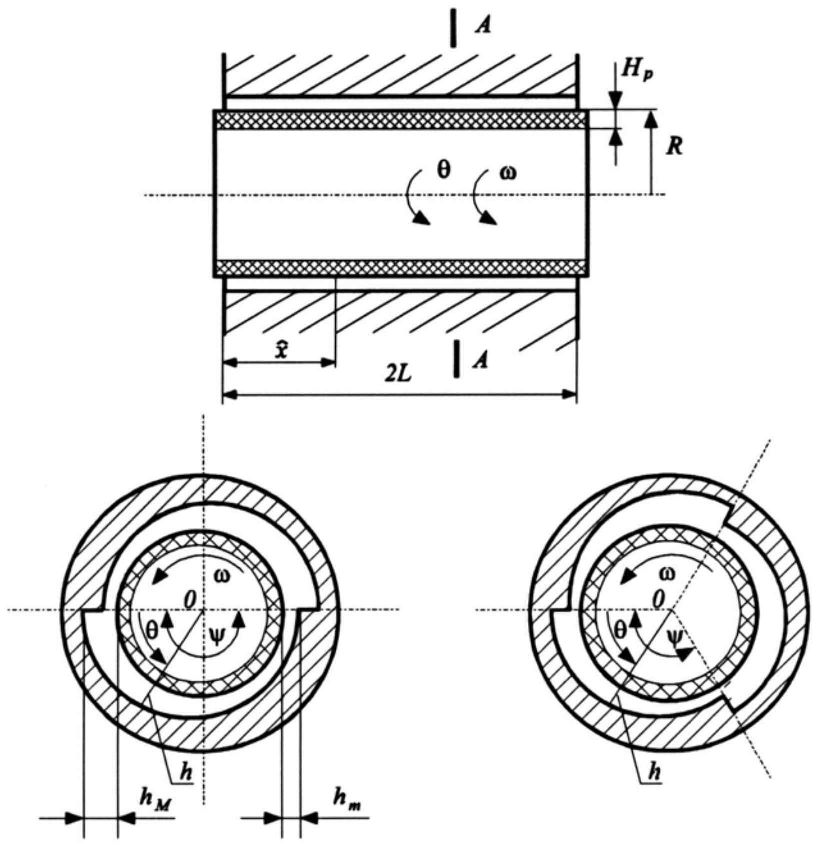

Fig. 1. Geometry of a multilobe conical bearing
Rys. 1. Geometria wieloklinowego łożyska

The bearing shaft rotates with angular velocity ω. The thickness of the bearing clearance is described by function *h(M, θ)* which denotes the distance between the sleeve surface and the outer surface of porous layer, measured along a normal to the fixed surface. The porous layer of thickness H_p (= constant) is connected with the rotating surface of the shaft.

In the case of a multilobe bearing discussed here the clearance thickness is described by the parameter M which is given by the ratio:

$$M = \frac{h_M}{h_m} \tag{1}$$

where h_m, denotes the minimum thickness of the clearance in an arbitrary cross-section of the bearing and h_M denotes the maximum thickness of this clearance.

Denoting by ψ the arc span of the lobe one can express the bearing clearance in the form:

$$h = h_M H(M, \psi, \Theta) \qquad \text{where} \qquad h_M = \text{const.} \tag{2}$$

Note that there is:

$$H(M, \psi, 0) = 1 \qquad \text{and} \qquad h = h_M, \qquad \text{for } \Theta = 0, \tag{3}$$

$$H(M, \psi, \psi) = \frac{1}{M} \qquad \text{and} \qquad h = h_m = \frac{h_M}{M}. \qquad \text{for } \Theta = \psi. \tag{4}$$

The thickness of the bearing clearance considered in the present paper is given by the function:

$$h = h_M\left(1 - \frac{M-1}{M}\frac{\Theta}{\psi}\right) \quad \text{and} \quad H = 1 - \frac{M-1}{M}\frac{\Theta}{\psi} \tag{5}$$

where $\psi = \frac{2\pi}{N}$ is the arc span of a single lobe.

REYNOLDS EQUATION

The Reynolds equation for an incompressible fluid and for a bearing with porous wall can be written as [1, 8, 11]:

$$\frac{1}{R}\frac{\partial}{\partial \hat{x}}\left[R\left(h^3 + 12\Phi H_p\right)\frac{\partial p}{\partial \hat{x}}\right] + \frac{1}{R^2}\frac{\partial}{\partial \Theta}\left[\left(h^3 + 12\Phi H_p\right)\frac{\partial p}{\partial \Theta}\right] = 6\mu\omega\frac{\partial h}{\partial \Theta} \tag{6}$$

where: p - is the hydrodynamic pressure,

μ - is the viscosity of lubricant fluid,

Φ - is the permeability of porous layer.

Define the following (dimensional or nondimensional) parameters:

$$\eta = \frac{L}{R} \qquad \Lambda = \frac{6\mu\omega R^2}{h_M^2}, \qquad P = \frac{p}{\Lambda},$$

$$K_p = \sqrt[3]{\frac{12\Phi H_p}{h_M^2}}, \qquad x = \frac{\hat{x} - L}{L} \tag{7}$$

where: K_p - is the parameter of porosity,

x - is the bearing coordinate varying in the interval: $\pm l$.

Introducing the above parameters, the Reynolds equation (6) can be presented in a dimensionless form:

$$\frac{1}{\eta^2}\frac{\partial}{\partial x}\left[\left(H^3 + K_p^3\right)\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial \Theta}\left[\left(H^3 + K_p^3\right)\frac{\partial P}{\partial \Theta}\right] = \frac{\partial H}{\partial \Theta}. \tag{8}$$

SOLUTION OF THE REYNOLDS EQUATION

The solution of Eq. (8) for the pressure distribution will be searched for in the form of a product [10]

$$P = P_\infty(\Theta) f(x) \tag{9}$$

where $P_\infty(\Theta)$ is the solution of the following equation:

$$\frac{\partial}{\partial\Theta}\left[\left(H^3+K_p^3\right)\frac{\partial P}{\partial\Theta}\right]=\frac{\partial H}{\partial\Theta} \tag{10}$$

describing the pressure distribution in the bearing clearance without an annular lubricant flux (then: $x=0, \frac{\partial}{\partial x}=0$ may be put).

Integrating Eq. (10) we obtain [9]

$$P_\infty(\Theta)=\int_0^\Theta \frac{H-H^*}{\left(H^3+K_p^3\right)}d\Theta \tag{11}$$

where H* is the lubricant film thickness when the pressure P_∞ attains its maximum.

To define the value of H^* one puts that $P(\psi)=0$, then the value of H^* is given by the formula:

$$H^*=\frac{\int_0^\psi \frac{H}{H^3+K_p^3}d\Theta}{\int_0^\psi \frac{1}{H^3+K_p^3}d\Theta}. \tag{12}$$

Assuming that $Kp \ll H$ the values of $P_\infty(\Theta)$ and H^* may be calculated analytically. The graphs of H* are presented in Fig. 2.

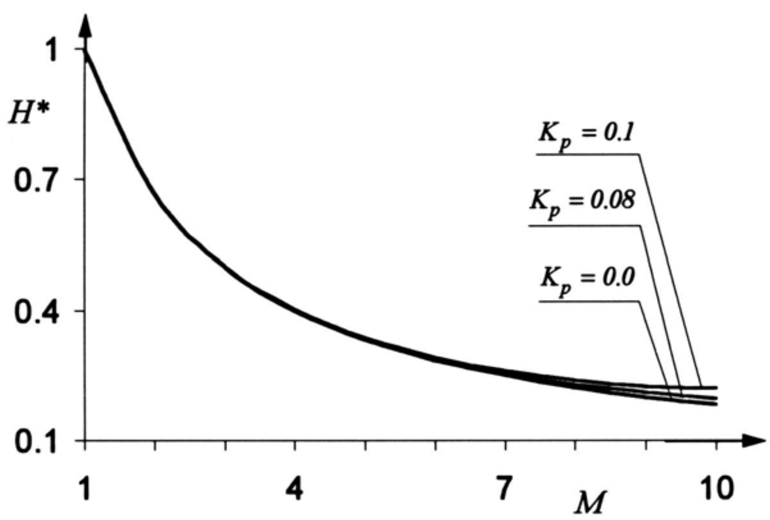

Fig. 2. Graphs of H^* versus M witki porosity coefficient K_p as parameter

Rys. 2. Wykresy H^* w funkcji M dla różnych wartości współczynnika porowatości K_p We will search the function f(x) in the form:

$$f(x)=\sum_{m=1}^{n} C_m f_m(x) \tag{13}$$

Using Galerkin's method to define the coefficients C_m each function $f_m(x)$ will be considered as the test function having the zeros at the points x = ±1 ; this condition is equivalent to the statement that the pressure is equal to zero on the bearing ring.

Galerkin's system of equations to determine the coefficients C_m has the form:

$$\int_0^{\psi}\int_{-1}^{+1}\left\{\frac{1}{\eta^2}\frac{\partial}{\partial x}\left[\left(H^3+K_p^3\right)\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial\Theta}\left[\left(H^3+K_p^3\right)\frac{\partial P}{\partial\Theta}\right]\right\}P_\infty(\Theta)f_i(x)d\Theta dx=\int_0^{\psi}\int_{-1}^{+1}\left(\frac{\partial H}{\partial\Theta}\right)P_\infty(\Theta)f_i(x)d\Theta dx \,. \tag{14}$$

Determining the integrals in Eq. (14) we have:

$$\sum_{m=1}^{n}\left[C_m\int_{-1}^{+1}f_m(x)f_i(x)dx\right]+G\frac{1}{\eta^2}\sum_{m=1}^{n}\left[C_m\int_{-1}^{+1}\frac{d^2f_m(x)}{dx^2}f_i(x)dx\right]=\int_{-1}^{+1}f_i(x)dx \tag{15}$$

where

$$G=\frac{\int_0^{\psi}\left[P_\infty(\Theta)\right]^2\left(H^3+K_p^3\right)d\Theta}{\int_0^{\psi}\left[P_\infty(\Theta)\right]\frac{\partial H}{\partial\Theta}d\Theta} \tag{16}$$

is the parameter of circumferential pressure distribution.

The form of coefficients C_m depends on the form of functions $f_m(x)$ approximating the pressure distribution along the bearing generatrix. It is convenient to take these functions in trigonometric shape of

$$f_m(x)=\cos\left(\frac{2m-1}{2}\pi x\right). \tag{17}$$

Upon substituting Eq. (17) finto Eq. (15) and calculating for *i=m* we have the following set determining the coefficients C_m

$$C_m=\frac{\frac{2}{a_m}(-1)^{m+1}}{1-\frac{a_m^2}{\eta^2}G}, \qquad \text{and} \qquad a_m=\frac{2m-1}{2}\pi \tag{18}$$

MECHANICAL PARAMETERS

In case of one-lobe journal bearing the Cartesian components of the radial load are given by:

$$W_x=\int_0^{2\pi}\int_0^{2L}pR\cos\Theta d\Theta d\hat{x}, \qquad W_y=\int_0^{2\pi}\int_0^{2L}pR\sin\Theta d\Theta d\hat{x} \tag{19}$$

Taking finto account relations (7) we can write:

$$W_{xj} = \frac{3\mu\omega R^3 L}{h_M^2}\int_0^{2\pi} P_\infty(\Theta)\cos\Theta d\Theta \sum_{m=1}^{n} C_m \int_{-1}^{+1} f_m(x)dx = P_{Rj} W_{x\infty} K_j , \tag{20}$$

$$W_{yj} = \frac{3\mu\omega R^3 L}{h_M^2}\int_0^{2\pi} P_\infty(\Theta)\sin\Theta d\Theta \sum_{m=1}^{n} C_m \int_{-1}^{+1} f_m(x)dx = P_{Rj} W_{y\infty} K_j . \tag{21}$$

where

$$P_{Rj} = \frac{3\mu\omega R_a^3 L}{h_M^2}, \qquad K_j = \sum_{m=1}^{n} C_{mj} \frac{2}{a_m}(-1)^{m+1},$$

$$W_{x\infty} = \int_0^{2\pi} P_\infty(\Theta)\cos\Theta d\Theta , \qquad W_{y\infty} = \int_0^{2\pi} P_\infty(\Theta)\sin\Theta d\Theta . \tag{22}$$

The friction torque *MTV* fis given by:

$$M_{Tj} = \int_0^{2\pi}\int_0^{L} \tau R^2 d\Theta d\hat{x} = \frac{R_a^2}{2}\int_0^{2\pi}\int_{-1}^{+1} \tau d\Theta dx . \tag{23}$$

Talcing finto account that:

$$M_T = \frac{\mu R\omega}{h} + \frac{h}{2R}\frac{\partial p}{\partial\Theta}$$

we have:

$$M_{Tj} = M_{1j} I_{1j} + M_{2j}\left(I_{1j} - I_{2j}\right) K_j \tag{24}$$

where

$$M_{1j} = \frac{\mu\omega R_a^3 L}{h_M}, \qquad M_{2j} = \frac{3\mu\omega R_a^3 L}{2h_M},$$

$$I_{1j} = \int_0^{2\pi} H^{-1} d\Theta, \qquad I_{2j} = H^* \int_0^{2\pi} H^{-2} d\Theta . \tag{25}$$

and K_j fis given by Eq. (22).

EXAMPLES OF APPLICATION

Let us consider a multilobe bearing (Fig. 1) for which the geometrical parameters are *LlR=3,* $\eta=1$.

Figures 3 and 4 show the pressure distribution P along the arc span of the lobe for twolobe bearing *(N = 2)* and for different values of *M.*

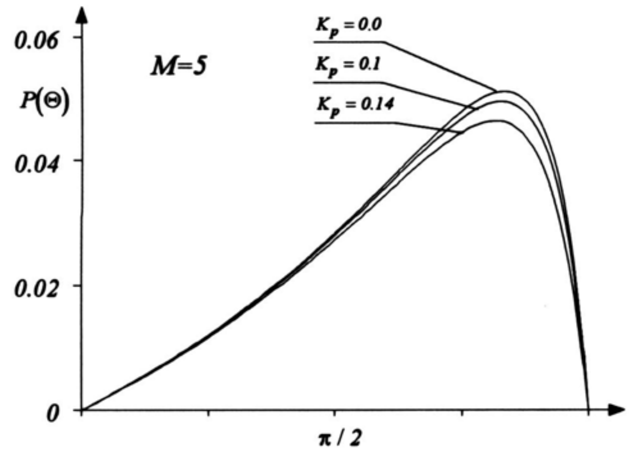

Fig. 3. The pressure distribution P(Θ) for two-lobe journal bearing witki clearance convergence *M=5* for diiferent values of porosity parameter K_p

Rys. 3. Rozkład ciśnienia P(Θ) dla dwuklinowego łożyska poprzecznego z parametrem zbieżności szczeliny M=5 dla różnych wartości współczynnika porowatości K_p

It is easy to see from the graphs presented in Figs 3-4 that the porosity of wall have a considerable influence on the pressure distributions; this porosity influences also on the mechanical parameters. For higher values of the parameter of the porosity K_p the pressure decreases considerably.

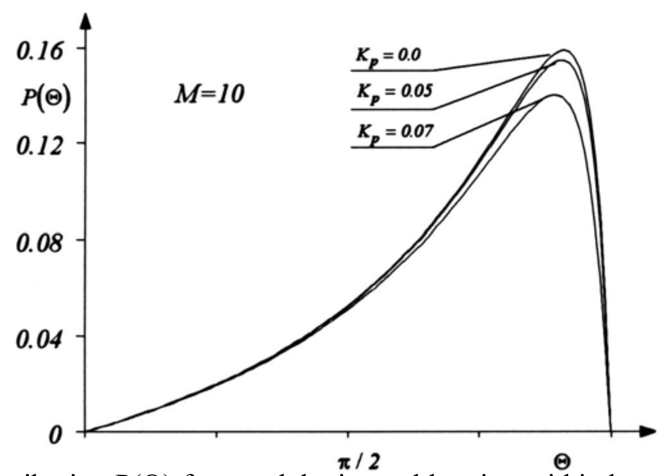

Fig. 4. The pressure distribution P(Θ) for two-lobe journal bearing witki clearance convergence *M=20* for different values of porosity parameter K_p

Rys. 4. Rozkład ciśnienia P(Θ) dla dwuklinowego łożyska poprzecznego z parametrem zbieżności szczelinyM=20 dla różnych wartości współczynnika porowatości K_p

CONCLUSIONS

From the proceeding calculations and their graphical presentations one may conclude that the pressure values decrease witki the increase of the parameter of porosity Kp and the parameter of lobe convergence M.

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PARAMETRY MECHANICZNE WIELOKLINOWEGO ŁOŻYSKA POPRZECZNEGO Z POROWATĄ ŚCIANKĄ

Streszczenie: W pracy są rozważane parametry mechaniczne wieloklinowego łożyska ślizgowego z porowatą ścianką wałka. Aby uzyskać rozwiązanie problemu autorzy wykorzystali metodę Galerkina przy założeniu, że łożysko smarowane jest nieściśliwą Newtonowską cieczą. Rozwiązanie otrzymano przy założeniu, że bezwymiarowy współczynnik porowatości ścianki spehua warunek $K_p << H$. Stwierdzono, że porowata ścianka zmniejsza ciśnienie w łożysku a co za tym idzie obniża parametry mechaniczne łożyska.

Recenzent: Prof. dr hab. inż. Jan Burcan